

THE PHILOSOPHY  
OF SYMBOLIC FORMS

VOLUME THREE: THE PHENOMENOLOGY OF KNOWLEDGE

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## Chapter I

### *Toward a Theory of the Concept*

#### 1

IF WE WISH to apply a single, over-all name to the field in which our investigation has moved so far, we may call it the realm of the "natural world concept." This realm disclosed throughout a very definite theoretical structure, an intellectual formation and articulation—but on the other hand the universal rules of this formation seemed so deeply and inextricably shot through with sensuous particulars that they could only be described in conjunction with them. At this stage of our investigation what theoretical form itself is and wherein consist its specific significance and validity could only be shown through its product. Its principles remained, one might say, fused with this product—they were not determined *in abstracto*, detached and in themselves, but could be demonstrated only through a certain order of objects, of objective structures of intuition. Here, accordingly, reflection and reconstructive analysis were not yet directed toward the function of form as such, but were directed toward one of its particular achievements. In forming a determinate image of objectivity, in producing it, so to speak, out of itself, thought nevertheless remains confined within this very image which originates from its own depths—knowledge of itself can come to it only through this medium, through the mediation of an objective knowledge. Its regard is directed forward toward the reality of things, not backward toward itself and its own achievement. In this way it conquers the world of the "thou" and the world of the "it"—and both appear to it at first as unquestionable, utterly unproblematic certainties. In the form of the simple expressive experience or of the perceptual experience, the I apprehends the existence of subjects and objects outside us—and lives in the concrete intuition of this existence. How this intuition itself is possible is not

asked here and need not be asked; it stands for itself and bears witness to itself, requiring no support or confirmation in anything else.

But this absolute trust in the reality of things begins to be shaken as soon as the problem of truth enters upon the scene. The moment man ceases merely to live in and with reality and demands a knowledge of this reality, he moves into a new and fundamentally different relation to it. At first, to be sure, the question of truth seems to apply only to particular parts and not to the whole of reality. Within this whole, different strata of validity begin to be marked off, reality seems to separate sharply from appearance. But it lies in the very nature of the problem of truth that once it arises it never comes to rest. The concept of truth conceals an immanent dialectic that drives it inexorably forward, forever extending its limits—it does not content itself with questioning particular contents of the natural world concept, but assails its substance, its general form. All those witnesses to reality that have hitherto been taken as absolutely sure and reliable—sensation, representation, intuition—are now haled before a new forum. This forum of the concept and pure thought is not established merely with the emergence of strictly philosophical reflection; it belongs to the beginnings of every scientific view of the world. For even here, thought no longer contents itself with simply translating into its own language what is given in perception or intuition, but subjects it to a characteristic change of form, a spiritual recasting. The primary task of the scientific concept seems in fact to be simply this, to set up a rule of determination which must be confirmed in the intuitive sphere. But precisely because and insofar as this rule is to have validity for the world of intuition, it is no longer a mere part or element of this world. Over against this world it signifies something peculiar and independent, even though this independent meaning can be manifested at first only through the matter of the intuitive world. The farther the scientific consciousness develops, the clearer this difference becomes. Now the rule of determination is no longer simply posited, but in this very positing is apprehended and understood as a universal achievement of thought. And it is this understanding that now creates a new form of insight, of spiritual perspective. With it we stand for the first time on the threshold of the actual theoretical world view. We have a classical example of this process in the genesis of Greek mathematics. For here the decisive factor is not that the basic significance of number is recognized, not that the cosmos is subjected to the law of number. This step had been

taken long before the beginnings of strictly theoretical, scientific thinking. Myth had already elevated number to a universal, truly world-embracing significance; myth already knew and spoke of its dominion over the whole of being, of its demonic omnipotence.<sup>1</sup> The first scientific discoverers of number, the Pythagoreans, were still wholly dominated by this magical-mythical view of number. And aside from this mythical fetter, their concept of number discloses another, purely intuitive tie. Number is not conceived as an independent entity but must always be considered as the enumeration of a concrete quantity. It is bound up most particularly with spatial determinations and spatial configurations; it is originally of a geometrical as well as arithmetical nature. But as this bond loosens, as the purely logical nature of number becomes recognized, the groundwork is laid for a pure science of number. Even then, to be sure, number is not separated from intuitive reality, for it aspires to demonstrate nothing less than the fundamental law governing this reality, governing the physical cosmos. But number itself ceases to resemble a physical thing in any way or to be definable by the analogy of any empirical objects. Although it has physical existence solely through concrete things that are ordered according to it, nevertheless one must attribute to it a form of knowledge that is clearly separate from sense perception or intuition. Solely by virtue of this separation was the number of the Pythagoreans able to become a genuine expression of the truth of the sensuous world.<sup>2</sup>

And this relationship that emerges in the beginnings of pure theory remains a determining factor in its further development. Time and time again it becomes apparent that theory can achieve the desired closeness to reality only by placing a certain distance between itself and reality, by learning more and more to look away from it. It is through this characteristic relegation to a distance that the *configurations* within which the natural world view dwells and through which it gains its formation are transformed into strict theoretical *concepts*. What lay like undiscovered treasure within the intuitive forms is now gradually brought to light by conscious intellectual effort. The prime achievement of the concept is precisely to apprehend as such the factors on which rests the

1. Cf. above, 2, 140 ff.

2. Concerning this twofold position of the Pythagorean number, see my "Die Geschichte der antiken Philosophie," in Max Dessoir, ed., *Lehrbuch der Philosophie* (Berlin, 1925), 1, 29 ff.

articulation and order of intuitive reality and to recognize their specific significance. It develops the relations which in intuitive existence are only posited implicitly, as merely concomitant; it detaches these relations and sets them forth in their pure "as suchness"—in Plato's words as an *αὐτο καθ' αὐτό*.

But with this transition to the realm of pure meaning, thought is confronted with an abundance of new problems and difficulties. For only now is the final break made with mere existence and its immediacy. Expression, and to an even greater extent representation, already reached out beyond this immediacy—for they did not remain within the sphere of mere presence but sprang from the basic function of signification. Yet it is within the sphere of pure meaning that this function not only increases in scope but first clearly discloses its specific direction. Now there develops a kind of detachment, of abstraction that was unknown to perception and intuition. Knowledge releases the pure relations from their involvement with the concrete and individually determined reality of things, in order to represent them purely as such in the universality of their form, in their relational character. It is not sufficient to construe being itself in the various directions of relational thinking, for knowledge also demands and creates a universal system of measurement for this procedure itself. As theoretical thinking progresses, this system is more and more firmly grounded and is made more and more inclusive. The naive relation between concept and intuition that characterizes the natural world concept is replaced by a new critical relation. For the theoretical concept in the strict sense of the word does not content itself with surveying the world of objects and simply reflecting its order. Here the comprehension, the "synopsis" of the manifold is not simply imposed upon thought by objects, but must be created by independent activities of thought, in accordance with its own norms and criteria. And whereas within the sphere of the natural world concept the activity of thought still shows a more or less sporadic character; whereas it begins now at this, now at that point from which it unfolds in different directions; here it takes on an increasingly full comprehensiveness and a strict and conscious concentration. All concept formation, regardless of the special problem with which it may start, is ultimately oriented toward one fundamental goal, toward determination of the "absolute truth." Ultimately thought seeks to fit all particular propositions, all particular conceptual structures into a unitary and all-inclusive intellectual context.

This task would not be possible if thought, in undertaking it, did not at the same time create a new organ for it. It can no longer content itself with the ready-made configurations that come to it from the world of intuition but must begin to build up a realm of symbols in full freedom, in pure spontaneous activity. It constructively draws up the schemata by which and toward which it orients the whole of *its* world. Of course these schemata cannot remain in the vacuum of sheer abstract thinking. They require a foundation and support—but this is no longer taken simply from the empirical world of things; it is created by thought. The system of relations and of conceptual significations is now provided with an aggregate of signs which is so constituted that the relationship prevailing between the various elements of the system can be surveyed and read from it. The farther thought advances, the closer this bond is drawn. And now it would actually seem to be one of the ideal goals of thought to provide every combination among the contents toward which it is directed with a corresponding combination or operation with signs. The *scientia generalis* now calls for a *characteristica generalis*. In this characteristic the work of language continues; but at the same time it enters into a new logical dimension. For the signs of the characteristic have cast off everything that is merely expressive or for that matter intuitive: they have become pure signs of signification. With this we have a new mode of objective relation that differs specifically from every kind of relation to the object that occurs in perception or empirical intuition. To apprehend the factors in this difference must be the first task of any analysis of the function of the concept. In every concept, whatever its particular character, there may be said to live and prevail a unitary will to knowledge, whose direction and trend as such must be determined and understood. Only when the nature of this universal form of the concept has been clarified, only when it has been sharply set off from the special character of perceptual and intuitive knowledge, can we progress to special investigations, advance from the function of the concept as a whole to its particular manifestations and workings.

## 2

The analysis of intuitive knowledge has shown us that the form of intuitive reality rests essentially on the fact that the particular factors from which it is built up do not stand by themselves, but that a peculiar rela-

tion obtains between them, a relation of being "com-posed." Nowhere in the intuitive world do we find anything isolated and detached. Even what seems to belong to a definite and particular point in space, or to a single moment in time, does not remain confined to a mere here and now. It reaches out beyond itself—it points toward the totality of the content of experience and joins with them into definite totalities of meaning. In the construction of every spatial intuition, of every apprehension of spatial forms, of every judgment as to the position, magnitude, distance of objects, the individual experiences "weave themselves into the whole." In order to be spatially determined in relation to the whole, every spatial content must be referred to and interpreted according to certain typical spatial configurations. These interpretations, as effected in the sign language of sense perception, may be regarded as primary achievements of the concept. For indeed they contain one factor that tends wholly in the direction of the concept and its truly fundamental achievement. They articulate the individual and particular into a determinate totality, and in the particular they see a representation of this totality itself. As intuitive knowledge progresses along this path, each of its particular contents gains greater power to represent the totality of the others and to make it indirectly "visible." If we take this representation as a characteristic determinant of the function of the concept as such, there can be no doubt that the worlds of perception and spatial-temporal intuition can nowhere dispense with this function. In the modern theory of perception this view was upheld primarily by Helmholtz, who made it the basis for the whole structure of his physiological optics. "If 'to understand' means to form concepts," he writes,

and if in the concept of a class of objects we sum up the like characteristics they disclose, it follows analogously that the concept of a number of phenomena which changes in time must seek to embrace what remains the same in all their stages. What remains the same without dependency on anything else through all the changes of time, we call *substance*; the unchanging relation between variable magnitudes we call *law*. It is only the latter that we perceive directly. . . . The first product of the intellectual understanding of phenomena is the *lawful*. . . . What lies within our reach is knowledge of the lawful order in the realm of the real, and this to be sure only as represented in the sign system of our sense impressions.



In this view the logical concept accomplishes nothing other than to fixate the lawful order that already lies in the phenomena themselves: to establish consciously the rule which perception follows unconsciously. In this sense Helmholtz holds, for example, that our mere intuitive idea of the stereometric form of a physical object entirely fills the role of a concept condensed from a large number of sensuous intuitive images. This concept however is not necessarily held together by determinations expressible in words such as a geometrician might construct, but only by the living idea of the law in accordance with which the manifold perspective-images of this particular physical thing follow one another. Hence the representation of an individual object must already be designated as a concept, because this representation "embraces all the possible aggregates of sensation which this object can call forth when regarded, touched, or otherwise examined from different sides."<sup>3</sup>

Helmholtz himself saw and stressed that this view, which places the function of the concept in the midst of the perceptual process itself, accords neither with the usual linguistic usage nor with traditional logic. The logical tradition finds the true and salient characteristic of the concept in its universality, and it regards the universal as that which is common to many. But how can such a community prevail where we are not comparing one object with others but, rather, constituting, acquiring the idea of an individual object? Yet Helmholtz would have been justified in rejecting this argument, for on closer scrutiny it contains within it a *petitio principii*. Precisely that universality which is here regarded as the necessary condition of the concept signifies not so much a secure result of logical analysis as a latent postulate to which logic as formal logic has been subordinated since its beginnings. The modern development of logic has increasingly shown the questionable nature of this very postulate. The notion that the concept must necessarily embrace the idea of a "class" and that all relations that can obtain among concepts must ultimately be reducible to a single fundamental relation of subsumption, of subordination of genera and species, has been combated from the most diverse angles by modern logicians.<sup>4</sup> If we abandon this view and with Kant interpret the concept as nothing other than the unity of rule by which a

3. Helmholtz, *Handbuch der physiologischen Optik*, pp. 599 ff., 948.

4. Cf. Wilhelm M. Wundt, *Logik. Eine Untersuchung der Prinzipien und der Methoden Wissenschaftlicher Forschung* (2d ed. Stuttgart, 1893-95), 1, 99 ff. Christoph Sigwart, *Logik* (2d ed. 2 vols. Freiburg, 1889-93), 1, 319 ff.

manifold of contents are held together and connected with one another—then it is clear that the structure of our perceptual or intuitive world cannot dispense with such a unity. For only through this unity do determinate configurations stand out within intuition itself; only through it are stable affinities created by virtue of which manifold and qualitatively different phenomena are taken as properties of one and the same object.

What is decisive here is obviously not that a common factor is detached from the phenomena and that they are subsumed under a universal idea, but that they fulfill a common function—that precisely in their thoroughgoing diversity they are oriented toward and indicate a definite goal. But the form of this indication is of course different in the sensory-intuitive world and in the world of the logical concept in the restricted sense. For the indication that is merely *employed* in perception or intuition becomes *conscious* in the concept. It is this new mode of awareness that first truly constitutes the concept as a form of pure thought. Even the contents of perception and pure intuition themselves cannot be thought as *determinate* contents without a characteristic form of determination—without a viewpoint, under which they are placed and in regard to which they are looked upon as belonging to one another. But the regard of perception or intuition rests on the elements which are compared or in some way correlated, not on the manner, the mode, of the correlation. It is with the logical concept that this mode of correlation first emerges. It first effects that reversal by which the I turns from the objects apprehended in vision, toward the manner of the seeing, the character of the vision itself. Only where this specific mode of reflection is practiced do we stand in the true realm of thought, and in its center and focus. And this is the source of the rich significance that the concept assumes within the problem of symbolic formation. For now this problem appears in a different aspect and enters into a new logical dimension.

The dividing line between intuition and concept is usually drawn so as to distinguish intuition as an immediate relation to the object from the mediated discursive relation of the concept. But intuition itself is discursive in the sense that it never stops at the particular but strives toward a totality it never achieves in any other way than by running through a manifold of elements and finally gathering them into one regard. Yet over against this form of intuitive synthesis the concept establishes a higher potency of the discursive. It does not simply follow

the fixed directives provided by the similarity of phenomena or by any other intuitive relation between them—it is no ready-made path but a function of pathfinding itself. Intuition follows set paths of combination, and herein consists its pure form and schematism. The concept, however, reaches out beyond these paths in the sense that it not only knows them but also points them out; it not only travels a road that is opened and known in advance but also helps to open it.

It is of course this fundamental power of the concept that from the standpoint of a strict empiricism makes it seem to be tainted once and for all with subjectivity. This suspicion and reproach run through the whole positivist and empiricist theory of knowledge. It was the essence of Bacon's critique of conceptual thinking that it does not content itself with the reality of experience as something purely given—that, instead of solely receiving this reality, it transforms it in one way or another and so falsifies it. Thus the freedom and spontaneity of the concept are looked upon as sheerly arbitrary. But the profounder reason for this reproach is that empiricism fails to take this freedom itself in its full significance and scope but interprets it as a mere freedom of combination. In the empiricist view the concept can posit and produce no new content of knowledge; it can only transpose in various ways the simple ideas presented by sensation, and join them and separate them at will. Thus from the original data of knowledge it fashions derived phenomena which are mere products of mixture and accordingly have all the instability of such products. "Mixed modes"—so Locke formulates this fundamental view—arise wherever the understanding does not content itself with apprehending what is present in inward or outward perception but from it forms new connections which belong solely to itself. For these modes there are no prototypes, no originals either in sensation or in the world of real objects:

But if we attentively consider these ideas I call mixed modes, we are now speaking of, we shall find their original quite different. The mind often exercises an active power in making these several combinations: for it being once furnished with simple ideas, it can put them together in several compositions, and so make variety of complex ideas, without examining whether they exist so together in nature. And hence I think it is that these ideas are called notions, as if they had their original and constant existence more in the thoughts of men than in

the reality of things . . . Every mixed mode consisting of many distinct simple ideas, it seems reasonable to inquire, "whence it has its unity, and how such a precise multitude comes to make but one idea, since that combination does not always exist together in nature?" To which I answer, it is plain it has its unity from an act of the mind combining those several simple ideas together, and considering them as one complex one, consisting of those parts; and the mark of this union, or that which is looked on generally to complete it, is one name given to that combination.<sup>5</sup>

This meager recognition of the concept in Locke's system of empiricism places it of course on so narrow and insecure a foundation that the very first attack will suffice to shatter its whole substance and validity. Here Berkeley proceeds more acutely and consistently when he takes back even this limited concession—when he declares the concept to be not so much an independent source of knowledge as the source of all illusion and error. If the foundation of all truth lies in simple sensory data, only mere fictions can arise as soon as this foundation is left behind. In this verdict which Berkeley pronounces upon the concept as such, concepts of every kind and of every logical rank are included—in fact it is primarily directed against the very concepts that would seem to be "most exact," those of mathematics and mathematical physics. In this view concepts taken all together are not roads to reality, to the truth and essence of things, but roads away from it; they do not sharpen the mind but blunt it to the single true reality that is given us in immediate perceptions.

Yet in this most radical rejection of the concept, we may say that from both the historical and the methodological point of view a peculiar reversal of thought is in preparation. Berkeley supposed that by his critique he had struck the concept in its root; but if we carry his critique to its conclusion, a positive factor results that is highly fruitful for the understanding and appreciation of the concept. For it is not the concept as such that Berkeley destroys; what he destroys at one stroke is rather the connection between the concept and the general idea which had hitherto been upheld by a centuries-old logical and psychological tradition. It is this that Berkeley resolutely set aside and recognized as an inner contradiction. The general idea, the image of a triangle that is not right-angled, acute-angled, or obtuse-angled but is all these at once, is an empty fiction. Yet in combating this fiction Berkeley, contrary to his own basic

5. John Locke, *An Essay Concerning Human Understanding*, Bk. II, chap. 22, secs. 2-4.

purpose, prepared the way for another and deeper view of the concept. For he, too, with all his opposition to the general idea, leaves the universality in the form of the representative function intact. A single concrete, intuitive image, a triangle with a definite magnitude of sides and angles, can despite its concrete character stand for all other triangles, can represent them for the geometrician. Thus from the intuitive idea of a triangle there arises its concept—and this does not mean that we simply obliterate certain determinations that are contained in it but that we posit them as variable. What holds together the various structures which we regard as examples of one and the same concept is not the unity of a generic image but the unity of a rule of change, on the basis of which one example can be derived from another and so on up to the totality of all possible examples. In rejecting the unity of the generic image, Berkeley does not contest the “unity of the rule.”<sup>6</sup>

But now one must ask whether and to what extent this admitted unity can be grounded in a pure psychology of ideas. The rule remains valid, although the manner of its validity cannot be made visible in any concrete image, in any direct perception. Consequently, when Berkeley seeks some sort of intuitive substrate for it, he must have recourse to the word, the name. Yet this nominalism does not solve the problem of the concept but only moves it one step backward. For the name becomes a name only through its power to designate and signify something. To take away this function is to take away its whole character as a name and reduce it to a mere sensuous sound; but if this function is recognized, the riddle of nominal signification merely revives the whole riddle of the concept. Instead of approaching the problem of signification indirectly through the name, we must move it into the center and focus of our investigation; we must inquire into the meaning of precisely that power of representation, of “standing for,” which even empiricism and the empiricist critique could not help acknowledging in the concept.

Here the most obvious method would seem to be to explore this basic relation by reducing it to a quantitative relation. The definition of the concept as the “one in the many” seems in itself to call for a quantification of this sort. This definition goes back to the very beginnings of concept thinking, to the discovery of the concept in Socratic induction and the Platonic dialectic. Ever since then it has been a classical tenet of logic and of philosophy in general. Kant, too, in distinguishing the con-

6. In regard to this positive core of Berkeley's theory of the concept, see the account of this theory in my *Erkenntnisproblem* (3d ed.), 2, 297 ff.

cept from intuition, defines it as a representation which is contained in an infinite number of different possible representations as their common characteristic, and which therefore encompasses these representations in subordination to itself.<sup>7</sup> And the surest if not the only way to define this characteristic, to ascertain its significance, would seem to be to effect the *discursus*, actually to run through the quantity from which the common factor is expected to stand out. We simply set the elements of this quantity side by side and by merely counting through them we shall immediately find the form of their unity; in them and through them we shall apprehend the logical bond that holds them together. A sensationalist psychology of the concept cannot but adopt this method, since for this sensationalism the unity of the concept, like the unity of the ego, breaks down into a mere bundle of ideas.

But the same reduction is demanded and favored by still another camp from what would seem to be a diametrically opposite point of view. The more logic tended toward mathematics, the more it strove to grasp the content of a concept through its extension, and ultimately to replace content by extension. For only insofar as this could be done did the aim of mathematical logic—a quantitative analysis of the qualitative factors of the concept—seem attainable. The concept seemed to be accessible to exact quantitative inquiry only if it were defined as an aggregate in the strict sense—only, that is to say, if it were taken as a class of elements forming none other than a purely collective unity. Only in this way, it was supposed, could logic take the step which natural science had taken long before, and through which it had first been raised to the rank of strict cognition. The homogenization of logic was achieved: the mutual relation and determination of concepts had been reduced to the basic rules of a calculus of classes. In this sense Schröder in particular, in his “Algebra der Logik” [below, note 9], sought to build up a pure logic of classes. Such a logic asks only whether or not classes fall within one another and considers the class as an aggregate of the elements it includes. What links these elements together is a mere *and*-relation: a relation which, according to Russell, may equally well connect a teaspoon with the number three and a chimera with four-dimensional space.<sup>8</sup> Yet even among mathematical logicians grave critical objections

7. Cf. *Kritik der reinen Vernunft*, 2d ed., p. 40.

8. Cf. Bertrand Russell, *Principles of Mathematics*, Cambridge, 1903; (2d ed. New York, W. W. Norton, 1938), p. 71.

were soon raised against this view of the concept. No less a logician than Frege argued in opposition to Schröder that the calculus of classes, whose fundamental relation is that of the part to the whole, must be regarded as wholly distinct from logic. "Indeed," he wrote, "I hold that the concept logically precedes its extension and regard as a fallacy any attempt to base the extension of the concept as a class not on the concept but on particular things. In this way one may arrive at a calculus of classes but not at a logic." Here the relation between mathematics and logic is seen and grounded in a fundamentally different direction than in Schröder: the connection between the two is sought not in the concept of classes but in the concept of the function, and the concept itself is essentially understood and defined as a function.<sup>9</sup>

The modern logic of mathematics has taken this view into account: even where it has adhered to the basic concept of the class and to the presuppositions of the calculus of classes, it has recognized, side by side with the calculus of classes, a wholly independent brand, a calculus of relations. In Russell's treatment of the principles of mathematics the concept of relation gradually achieves logical primacy over the concept of class. In the *Principles*, appearing in 1903, he wrote:

A careful analysis of mathematical reasoning shows . . . that types of relations are the true subject-matter discussed, however a bad phraseology may disguise this fact; hence the logic of relations has a more immediate bearing on mathematics than that of classes or propositions, and any theoretically correct and adequate expression of mathematical truths is only possible by its means. . . . it has always been customary to suppose relational propositions less ultimate than class-propositions (or subject-predicate propositions, with which class-propositions are habitually confounded), and this has led to a desire to treat relations as a kind of classes.<sup>10</sup>

Once relation has thus been recognized as the basic and essential factor in mathematical concepts and concepts in general, the attempt to explain

9. That Frege himself did not adhere strictly and consistently to his own basic view, but replaced it by a purely quantitative view of the concept, has been aptly shown by Wilhelm Burkamp in his *Begriff und Beziehung, Studien zur Grundlegung der Logik* (Leipzig, 1927), cf. esp. the fourth study, "Klasse und Zahl in der Begriffslogik." The quotation from Gottlob Frege [Friedrich Ludwig Gottlob] is to be found in his "Kritische Beleuchtung einiger Punkte in E. Schröder's Vorlesungen über die Algebra der Logik," *Archiv für systematische Philosophie*, new ser. 1 (1895), 433-56. Cf. Burkamp, p. 198.

10. Russell, *Principles*, chap. 2, sec. 27.

the content of a concept by its extension becomes untenable. Russell himself, it is true, goes on to define the concept purely as a class of elements; but in so doing he is compelled to distinguish between two definitions of classes. There are, he stresses, two ways to determine classes: one by pointing out their members one by one and connecting them as a mere aggregate, by a simple "and"—the other by stating a universal characteristic, a condition which all members of the class must fulfill. Russell sets this latter generation of the class, the "intensional," over against the former, which is explained by means of "extension." And they do not remain in such juxtaposition, for it becomes gradually clearer that the definition by intension has precedence over the definition by extension. First of all it has the advantage of greater logical universality, for it alone makes it possible to include classes embracing an infinite quantity of elements. Russell, it is true, seems at first to minimize this difference as "purely psychological." "Classes," he declares,

may be defined either extensionally or intensionally. That is to say, we may define the kind of object which is a class, or the kind of concept which denotes a class: this is the precise meaning of the opposition of extension and intension in this connection. But although the general notion can be defined in this two-fold manner, particular classes, except when they happen to be finite, can only be defined intensionally, *i.e.* as the objects denoted by such and such concepts. I believe this distinction to be purely psychological: logically, the extensional definition appears to be equally applicable to infinite classes, but practically, if we were to attempt it, Death would cut short our laudable endeavour before it had attained its goal.<sup>11</sup>

But, as far as I can see, Russell's logic was not able, in its development, to sustain this equivalence. More and more, the definition by intension assumes, in this logic, not only a subjective but also an objective priority—and comes to represent not only a *πρότερον πρὸς ἡμᾶς* but also a genuine *πρότερον τῇ φύσει*. For it is evident that before one can proceed to group the elements of class and indicate them extensively by enumeration, a decision must be made as to which elements are to be regarded as belonging to the class: and this question can be answered only on the basis of a class concept in the "intensional" sense of the word. What seems to hold together the members united in the class is that they

11. *Ibid.*, chap. 6, sec. 71.



all meet a certain condition which can be formulated in general terms. And now the aggregate itself no longer appears as a mere sum of individuals, but is defined by this very condition, whose meaning we can grasp and state by itself, without having to ask in how many individuals it is realized, or even whether it is realized in any individual at all. "When I pronounce a sentence with the grammatical subject 'all men,'" Frege had argued against Schröder, "I do not by this mean to say anything about an unknown chief in Central Africa. Thus it is absolutely false to say that in using the word 'man' I am in any way designating this chief." It is in line with the same fundamental view when Russell himself, in his *Principia Mathematica*, expressly declares that an extension is an *incomplete symbol, the use of which takes on meaning only through its relation to an intension*.<sup>12</sup> What holds the class together, according to the theory developed here, is the circumstance that all the members united in it are thought of as variables of a determinate propositional function: it is therefore this propositional function and not the mere idea of a quantity as a pure collective that becomes the core of the concept.

Here the propositional function as such must be strictly distinguished from any particular proposition, from a judgment in the usual logical sense. For what it primarily gives us is only a pattern for judgments but in itself is no judgment: it lacks the decisive characteristic of a judgment, since it is neither true nor false. Truth or falsity attach only to the individual judgment in which a definite predicate is related to a definite subject; whereas the propositional function contains no such definiteness but only sets up a general schema which must be filled with definite values before it can achieve the character of a particular statement. "A propositional function," Russell defines, "in fact, is an expression containing one or more undetermined constituents, such that, when values are assigned to these constituents, the expression becomes a proposition. In other words, it is a function whose values are propositions." In this sense every mathematical equation is an example of such a propositional function. Let us take the equation  $x^2 - 2x - 8 = 0$ . This statement is true if for the wholly indefinite value of  $x$  we substitute the two roots of the equation; for all other values it is false.<sup>13</sup> On the basis

12. Alfred North Whitehead and Bertrand Russell, *Principia Mathematica*, Cambridge, 1910; (2d ed. 1925), 2, 75. Cf., for greater detail, Burkamp, pp. 186 f.

13. Cf. Bertrand Russell, *Introduction to Mathematical Philosophy*, 1919; (2d ed. London and New York, Macmillan, 1920), pp. 155 f.

of these determinations we may give a general purely "intensional" definition of class. If we consider all  $x$ 's so constituted that they belong to the type of a certain propositional function  $\phi(x)$  and group together the values of  $x$  which prove to be "true" values for this function, we have defined a determinate class by means of the function  $\phi(x)$ . In this sense every propositional function yields a class, namely the class of  $x$ 's constituted so that they are  $\phi(x)$ —and precisely this "so that" cannot be broken down into other determinations, but must be recognized as a meaning *sui generis*, a "logical indefinable." Each class becomes definable only through the statement of a propositional function which is true for the members of that class and false for all other things.<sup>14</sup> But with this, what logic calls a concept has by no means been broken down into a collective quantity; on the contrary, the quantity is once again grounded in the concept. Here, accordingly, pure logical calculus has led us no further: it becomes apparent that it cannot replace the pure analysis of meaning—can never do any more than state it in the strictest and simplest formula.

Here, then, mathematics may be expected to furnish an analytical clarification but no truly genetic definition of the concept; yet in another respect it seems able to help logic toward such a definition. For although, as Kant argued in his methodology of pure reason, philosophy can expect no salvation from an "imitation of the methods" of mathematics, nevertheless mathematics provides it with those contents through which the specific meaning of the pure function of the concept can most clearly be discerned and most adequately apprehended. The concept seems to stand out with full clarity only in its exact mathematical formulation: here and only here do we seem to find, written in bold letters, what it is, signifies, and achieves. I myself took this path in an earlier investigation: by the example of mathematical and mathematical-physical concepts I attempted to arrive at a universal definition of the function of the concept. Of course it can be argued that such an approach takes the part for the whole; that a truly logical and phenomenological analysis of the concept must attempt to apprehend it in the totality of its meaning, in all its achievements and phases of achievement, whereas mathematics and exact science disclose it to be sure in perfection but by that very token only at the end of its development. Must not this end, the argument runs, be-

14. Cf. Russell, *Principles*, chap. 7, secs. 80, 84; *Introduction to Mathematical Philosophy*, chap. 17, pp. 181 ff.

linked with the beginning, and must we not survey and pass through all mediate and intermediate stages if we are to arrive at an exhaustive definition of the concept?

Actually some logicians have gone so far as not only to distinguish what they called the "logical concept" from the "scientific concept" but actually to regard them in a sense as polar opposites. According to Wundt, the logical and the scientific concept form the opposite ends in the development of thought, since it begins with the logical concept and concludes every determinate line of activity with the scientific concept. The logical concept is bound by only two fundamental conditions: its content must be determined and it must stand in a logical connection with other concepts; the scientific concept presents an additional requirement: in it knowledge must have arrived at a certain, at least relative, conclusion, and it must have justified its validity on all sides and so raised itself to the level of universality.<sup>15</sup> Consequently, any attempt to derive the structure of the logical concept as such from that of the scientific concept seems to amount to a confusion of genus and species. And what makes it harder to escape the force of this objection is that one of the most important findings of our present inquiry is precisely that it is necessary to recognize certain types of spiritual formation which, though differing sharply in character from the form of the scientific concept, cannot dispense with intellectual determinateness.<sup>16</sup>

Must this insight not react on our conception of logic itself—must we not expect to find here, too, a complex and differentiated totality of forms of thought and knowledge, instead of a single unitary type of "concept as such"? Indeed, our inquiry up to now has repeatedly shown us that what we called the symbolic formation of the perceptual and intuitive world does not begin with the abstract concept, and certainly not with one of its highest expressions, the concept of exact science. In order to understand this mode of formation and its fundamental direction, we had to begin our inquiry at a much lower level—with the natural world concept. But this step backward led us to a further truth which, far from altering our previous analysis of the exact concept, confirmed it from a new angle. For we found that the extended scope we were compelled

15. Wundt, *Logik*, 2d ed., 1, 95 ff. A similar conception has recently been put forward by Gerard Heymans, "Zur Cassirerschen Reform der Begriffslehre," and my counterremarks, both in *Kant-Studien*, 33 (1928), 109–28, 129–36.

16. Cf. my *Die Begriffsform im mythischen Denken*, *Studien der Bibliothek Warburg*, 1 (Leipzig, 1922).

to give our problem made no change in the pure meaning which we had sought to clarify by the example of mathematics with its relational concepts. Wherever we attacked this problem, whether at the highest or lowest levels of knowledge, whether we inquired into intuition or pure thought, into linguistic or logical-mathematical concept formation—in every case we found that logical “one in many” which appeared with identical meaning in the most diverse stages of concretion. And in all these cases this “over-arching” One is not so much a unity of the genus under which the species and individuals are subsumed as a unity of the relation by which a manifold is determined as inwardly belonging together.

Outstanding mathematicians have designated this fundamental form of relation as the core of the numerical concept and hence of mathematical thinking,<sup>17</sup> but it is by no means limited to this realm. It is at work both in the smallest and the largest phenomena: it dominates the whole of knowledge from the simplest sensuous “remarking” and recognition up to those supreme intellectual conceptions in which thought transcends everything that is given, in which, surpassing the mere actuality of things, it establishes its free realm of the “possible.” It is here, consequently, that the “concept” must be grounded and anchored. For a close logical and epistemological analysis, “to conceive” and “to relate” always prove to be correlates, genuine reciprocal concepts.<sup>18</sup> This correlation as such remains in force, regardless of which world concept we move in; regardless of whether we are dealing with the empirical “things” of our perceptive and intuitive world, with the hypotheses of natural science, or with the constructions of pure mathematics. Here the content of what is thought does not affect or alter the pure form of thought any more than, in Descartes’ well-known metaphor, the light of the sun is changed by the various objects it illumines. For the building of a “world”—whether it is

17. Cf. Dedekind, *Was sind und was sollen die Zahlen?* (see above, p. 257).

18. This thesis I put forward and justified in detail in *Substanzbegriff und Funktionsbegriff* (Berlin, 1910). Eng. trans. by William C. and M. C. Swabey, *Substance and Function* (Chicago, 1923). It is confirmed in all essential points by the most recent investigations of the problem of the concept, contained in Burkamp’s *Begriff und Beziehung*. In a penetrating critique of the theories of Schröder, Frege, and Russell, Burkamp definitely takes the step from a mere logic of classes to a pure logic of relation. For him, too, the intellectual functions of positing, identity, difference, and relation that form the basic presupposition for the form of number are the presupposition for all pure form in general: “They are the deeper foundation on which form of any kind can first be built up” (Burkamp, fourth study, sec. 86; fifth study, secs. 95 ff.).

taken as an aggregate of sensuous or logical, of real or ideal objects—is possible only in accordance with definite principles of articulation and formation. And the concept does nothing other than to separate out these formative factors and fixate them for thought. It sets up a definite direction and norm of *discursus*: it indicates the point of view under which a manifold of contents, whether belonging to the field of perception, intuition or pure thought, are apprehended and “seen together.” The ultimate reason for all the logical and epistemological disputes over the nature of the concept is that it was taken not thus, as a pure viewpoint, but as a visible thing, a something that was supposed to have its home in the sensory world, side by side with it or above it.

The two parties that confronted each other in this battle of the giants have erred in the same sense: the one by striving to seize the concept as though in their hands, the other by assigning it to a suprasensory abode while still regarding it as something substantial that exists precisely in this place. It is characteristic that where Plato comes closest to a knowledge of the pure relational nature of the concept, where he deepens the original form of his theory of ideas by insistence on a *κοινωνία τῶν γενῶν*, he rejects both conceptions—that here, in the *Sophist*, he is driven to attack both the concept blindness of the sensationalists and materialists and the conceptual realism of the “friends of ideas.”<sup>19</sup> But even the counter movement against this conceptual realism, the “nominalism” of the Middle Ages and modern times, is itself by no means free from the fetters it scoffs at. For where it attempts to determine the nature of the concept, it, too, is fundamentally grappling with shadows. Failing to find the concept as a thing, it makes it into a mere sound, a *flatus vocis*. But it also treats this sound, this word of language, like a kind of existence, though of a secondary nature, instead of setting off the pure function of signification in it and grounding its objective meaning precisely therein. Time and time again, materialists and spiritualists, realists and nominalists reach back into some sphere of being when they attempt to ascertain and hold fast the meaning of the concept. But right there they lose the deeper insight into the symbolic content either of language or of knowledge, for this insight is that no being is tangible or accessible except through meaning. Hence if we wish to conceive of the concept itself, we must not attempt to clutch it like an object.

At this point the inner contradiction in the sensationalist theory of

19. Cf. Plato, *Sophist*, 245E ff.

knowledge is most clearly revealed. Some idealist logicians have supposed that they could leave the world of appearance, the world of the senses, to the sensationalist view, in order the more securely to defend the intellectual world from all admixture with the sensuous and confirm it as an independent sphere subject to its own laws. Our fundamental problem, however, has led us along the opposite path from the very first: it has shown us more and more plainly that sensationalism is unable to gain a unitary and uncontradictory view even of the sensuous world. We were impelled to meet the sensationalists in a field which from time immemorial they have claimed as their own inalienable domain—to discredit their theory from the standpoint not of the idea but of the sensory phenomenon itself. For the analysis of sensory appearance showed that its very *appearing*, its presentation is impossible without an ordered and articulated system of purely representative functions. Before the aggregate of the visible could be constituted as a whole, as the totality of an intuitive cosmos, it required certain basic forms of vision which, though they may be disclosed through visible objects, cannot be confounded with them, and cannot themselves be taken as visible objects. Without the relations of unity and otherness, of similarity and dissimilarity, of identity and difference, the world of intuition can acquire no fixed form; but these relations themselves belong to the makeup of this world only to the extent that they are *conditions* for it, and not parts of it.

This relationship that disclosed itself to us in the basic and primal stratum of intuitive knowledge finds its confirmation when we go on to other and "higher" levels of thinking and understanding. Here the world of pure meaning adds nothing new in principle to the world of representation, but only unfolds what is already potentially contained in this world. On the other hand, to be sure, this progress from potency to act is the most difficult achievement of knowledge. For now knowledge must free the functions of "indication," contained in the forms of intuitive reality, from this containment and apprehend them purely as modes of functional validity. A theory of this validity is demanded: a theory of forms which on the one hand isolates the various kinds of relation that already prevail in the intuitive world and are here demonstrable *in concreto*; and on the other hand apprehends them in their mutual determinacy, their interdependence. Thus we have seen, for example, that definite theoretical norms prevail in the structure of the spatial world

and that this structure is made possible only because the various spatial perceptions continuously orient themselves by certain basic forms.<sup>20</sup> But it is geometrical knowledge that first apprehends the law to which these forms are subject and expresses it as such with objective determinacy. Here again the theory of the concept must avoid confusing the form of determination with the contents which through it are first made determinable: it must avoid confusing the law with what is subject to it. Though brought into a thoroughgoing relation with each other, the two must remain sharply separate in their meaning. Here the symbolic language of logical calculus can come to the help of an analysis of meaning, for in a sense such a logical calculus places the intellectual distinction here involved immediately before our eyes. If we conceive of the concept as defined, not by a listing of what falls under it, but purely intensionally by the indication of a definite propositional function, this propositional function  $\phi(x)$  contains two factors which are obviously dissimilar. The universal form of the function as designated by the letter  $\phi$  stands out sharply against the values of the variable  $x$  which may enter into this function as true variables. The function determines the relation between these values, but it is not itself one of them: the  $\phi$  of  $x$  is not homogeneous with the  $x$  series,  $x_1, x_2, x_3$  etc. "It is to be observed," Russell stresses in his theory of the propositional function,

that according to the theory of propositional functions here advocated, the  $\phi$  in  $\phi x$  is not a separate and distinguishable entity: it lives in the propositions of the form  $\phi x$ , and cannot survive analysis. I am highly doubtful whether such a view does not lead to a contradiction, but it appears to be forced upon us, and it has the merit of enabling us to avoid a contradiction arising from the opposite view. If  $\phi$  were a distinguishable entity, there would be a proposition asserting  $\phi$  of itself, which we may denote by  $\phi(\phi)$ ; there would also be a proposition not- $\phi(\phi)$ , denying  $\phi(\phi)$ . In this proposition we may regard  $\phi$  as variable; we thus obtain a propositional function. The question arises: Can the assertion in this propositional function be asserted of itself? The assertion is non-assertibility of self, hence if it can be asserted of itself, it cannot, and if it cannot, it can. This contradiction is avoided by the recognition that the functional part of a propositional function is not an independent entity.<sup>21</sup>

20. Cf. above, pp. 155-8.

21. Russell, *Principles of Mathematics*, chap. 7, sec. 85.

In this familiar logical paradox, we once again run into a difficulty that has troubled logic from time immemorial and deeply affected the whole development of metaphysics, none other than the old problem of universals, which now confronts us in a new form. Regardless of how this problem has been solved—whether the universals were conceived as preceding or following particular things, or as contained in them—all these supposed solutions disclose the same fundamental fallacy. For a pure relation of meaning they substitute a relation such as exists between empirical things or events. For it is only between empirical things and events that a statement of “before” or “after,” “inner” or “outer” can be made. Nearly all the parties in this struggle over universals were destined to take these metaphors of before and after, inner and outer, for valid logical if not metaphysical determinations. But such metaphors can no longer deceive us once we have noted that the “universal” and the “particular” are distinguished from each other not in being but in meaning, and that a difference in dimensions of meaning can never be reduced to such differences as prevail between spatial and temporal dimensions, or be adequately expressed in terms of them. Of all the solutions here attempted the most relatively satisfactory still seems to be the one which sought the being of the universals in particular things: *universalia non sunt res subsistentes, sed habent esse solum in singularibus*.<sup>22</sup> For here at least the outward division is avoided; here, though the image be borrowed from space, the strict correlation, the reciprocal relation between the universal and the particular, is rightly maintained.

But this same correlation immediately involves new difficulties; for it is in danger of being confounded with the homogeneity of the factors that are related to each other. The conceptual universal then becomes a mere common factor, a something which though itself not an independent, new thing, expresses a similarity that is present in things. The significance of the universal now seems reducible to this category of similarity, of *similitudo*. But with this the significance of the concept as a purely relational concept suffers an unwarranted restriction: in the system of relations, similarity is only a special case which cannot be magnified into the type of the conceptual relation as such. It is not solely in respect to similarity that a manifold can be compared and grouped together: to this form of grouping we must juxtapose others, equal in rank, which are determined according to totally different criteria, through other modes of

22. Thomas Aquinas, *Contra gentiles*, 1, 65.



“respect.” And every such mode of respect, every relation  $R_1, R_2, R_3$  etc. may raise the same claim; each of them defines a fully legitimate concept.<sup>23</sup> In regard to the universal factor of meaning which the concept sets up and outlines, all the things that fall under it are not only similar but identical: in order to be thought of as special instances of a concept the particular examples must satisfy the whole concept, i.e. the totality of the conditions it embraces.

But this identity of the respect in which it is to be considered does not require the elements of a multiplicity that are to be grouped together by the concept to disclose any common content: the respect itself is not any kind of thing which can be wholly or partly contained in these elements, which by some sort of spatial analogy may be said to “lie” in them. Does the functional equation, for example, in any way lie in the various values of the variables that we can insert in it as “true values”? The equation of a plane curve may be designated as the concept of this same curve, for in it we have a propositional function that is true for all values of the coordinates of the points in the curve but false for other values.<sup>24</sup> Through this condition the different points in the curve are composed into a unity, which, however, designates no common factor in them except for that which consists in this form of correlation. Once the law of such a correlation is established, the totality of possible points in space immediately breaks down into two distinct classes in respect to it: the points which fulfill the relation stated in this law and those which do not fulfill it. What intuition apprehends as a particular form with certain spatial characteristics and attributes now seems to be reduced by intellectual analysis to a universal rule of correlation. And this not only is true for mathematical concepts but represents an essential feature of all true conceptual structures. For it is always the basic function of the concept to gather together—*συνάγειν εἰς ἓν* as Plato called it—what is dispersed in intuition, even things that are totally disparate from the standpoint of intuition, by establishing a new, ideal reference point for them. Particulars which had previously tended apart, order themselves according to this point of reference, and through this unity of direction a unity of essence is stamped upon them—though this essence is not to be taken ontically but logically, as a pure determination of meaning. The convergence by which

23. Cf. my more detailed remarks in *Substanzbegriff und Funktionsbegriff*, chap. 1, esp. pp. 18 ff.

24. Cf. Russell, *Introduction to Mathematical Philosophy*, p. 156.

their sensuous or intuitive heterogeneity is overcome does not signify that a substantial sameness or accord is disclosed in the elements of the manifold but that, however different they may be from one another, they are taken as factors of a context of meaning, that each in its own place and role constitutes the totality and function of this meaning.

If we take this view of the unity of the concept, we perceive of course that—to employ a term which Kant coined in a different context—it can never be anything other than a “projected unity.” For the concept only establishes a standpoint of comparison and correlation, but makes no statement of whether anything exists which meets the determination it provides. For this reason alone it is obvious that an adequate explanation can never be gained from a consideration of its mere extension, from a consideration of the particular or the particulars. For it is by no means certain that any particular corresponds to the unity it establishes, that any particular falls under it. In the way of handling mathematical logic that sought to reduce the concept to the class, the introduction and insertion of the zero class always raised special difficulties. The zero class was indispensable to any complete logical theory of the concept as well as of number; but on the other hand it was loaded down with paradoxes and contradictions for every purely extensional view. It was precisely these paradoxes that ushered in a change of attitude, that led Russell, for example, to regard a purely extensional view as inadequate and to round it out and deepen it with an intensional approach. Obviously a class that has no elements cannot be defined by indication of its elements—it can only be designated intensionally by virtue of a definite propositional function.<sup>25</sup> One of the limitations of the usual abstraction theory of the concept is that it must presuppose as given the elements from which the concept is supposedly built up, from which it is supposedly abstracted. If the concept is to bring out the common factor in a series of particulars, it must have them as distinct sensuous or intuitive realities, before it can stamp them with its own form. According to this theory, it can designate only what is—not what is not. And it is this postulate that stands at the beginning of all logic; it constitutes the fundamental idea of the Eleatic logic. But Parmenides is followed by Democritus and Plato, both of whom give to nonbeing a new justification and meaning, the former in the realm of physics, the latter in the field of dialectics. The system of knowledge, the

25. In Russell's logic the zero class is defined as the class of all  $x$ 's which satisfy any function  $\phi x$ , but which is false for all values of  $x$ : see *Principles*, chap. 2, sec. 25.

community of interlocking concepts—Plato's *Sophist* teaches—is not achieved until we resolve to recognize being and nonbeing as equally justified and equally necessary factors. Every single concept embraces, side by side with a statement about being, an abundance of statements about non-being; every "is" in a predicative sentence can be fully understood only if we conceive of an "is not" as correlative with it.<sup>26</sup> Indeed the concept cannot effect an ideal determination of the real as long as it remains exclusively within the confines of this reality. Its peculiar and supreme achievement requires that it progress from the contemplation of the real to that of the possible—and this it cannot do if it shrinks back from its opposite, the "impossible." The whole history of science teaches us how eminently important the conception of the "not-possible" can be and how in many cases it is precisely this conception that opens up a free survey of the realm of the possible and of its systematic formation and structuring. If the concept is a mere criterion of relation and correlation, it must be free also to link contradictories: precisely in order that through this link it may learn to recognize the contradiction and penetrate its foundation. Thus it is wholly fruitful and meaningful to form such a concept as that of the "regular decahedron"—for precisely the non-being that it contains within it opens up to thought a new insight into the being of the geometric world, the structure of space. We have said that the concept is not so much a ready-made path along which thought progresses as a method, a process of pathfinding. In this process thought can operate wholly on its own initiative; it does not bind itself to fixed goals that are already finished and given but sets up new goals and asks whether there is a path leading to them, and if so what path. In the language of symbolic logic this means that neither truth nor falsity is imputed to the propositional function in which the concept is grounded, that it remains open for the time being whether there are definite values of the variable  $x$ 's for which this function holds. Such a propositional function *intends* a definite meaning but does not yet *fulfill* it: it gives no fixed and ready answer but only establishes the direction of inquiry. But all knowledge must be preceded precisely by such a fixing of the question if a clear and secure answer is to be found. Until certain lines of aim are set up for knowledge—as is done in the concept—inquiry cannot begin, the valid relations in the realm of empirical as well as ideal being cannot be determined. Here it is characteristic that in the history of philosophy

26. Cf. Plato, *Sophist*, esp. 248E ff.

the concept itself first emerges in the form of a question. Aristotle designates Socrates as the "discoverer" of the universal concept. But this discovery in Socrates takes the form not of a new kind of knowledge, but of a kind of nonknowledge. The Socratic question concerning "what is" contains within it the method of Socratic induction of the *λόγοι ἐπακτικοί*. And so it remains true, even in highly developed knowledge, that each newly acquired concept is an attempt, a beginning, a problem; its value lies not in its copying of definite objects, but in its opening up of new logical perspectives, so permitting a new penetration and survey of an entire problem complex. Thus while among the basic logical functions the judgment closes and concludes, the concept, by contrast, has essentially the function of opening up. It throws out questions, the final decision of which falls to the judgment; it merely sets up an equation, the solution of which is expected from analysis of a definite ideal realm of objects or from advancing experience. In this sense a concept can be effective and fruitful for knowledge, long before it is itself exactly "defined," i.e. carried to a complete and definitive determination. For one of its essential tasks is not to let the problems of knowledge come prematurely to rest, but to keep them in a steady flux, by guiding them toward new goals which it must first anticipate hypothetically. Here again we find that the concept is far less abstractive than prospective; it not only fixes what is already known, establishing its general outlines, but also maintains a persistent outlook for new and unknown connections. It not only takes up the similarities or connections which experience offers it, but also strikes new connections; it is a free line stroke that must always be attempted anew if the inner organization of the realms both of empirical intuition and of the logical-ideal object is to be brought out clearly.

This makes it evident why every theory of the concept that seeks to explain it by purely reproductive tendencies and to reduce it to such tendencies must necessarily fail. In the field of intuition and the pure "representative function" this limitation has already proved impossible; even at these levels it was not possible to devise a theory of perception and empirical knowledge in general without invoking the aid of the "productive imagination" at every step. In the concept, the work of the productive imagination stands before us in an intensified form. Hence we fall into a misunderstanding of its sheer "whatness" as soon as we attempt to transform it into a sum of reproductions, a mere aggregate of remembered images. For simple phenomenological reflection tells us that

if we take the "concept" as it is immediately given, it appears as something totally different from the mnemonic image, as something very individual that is by no means interchangeable with it. We must go behind the sphere of consciousness, we must pass from pure logic and phenomenology to physiology if we wish to maintain an equivalence between concept and remembered image. The concept then becomes a product of unconscious traces and residues that have been left behind in the brain by earlier sense perceptions. But aside from the fact that this notion distorts the simple meaning of the logical question, transforming logic into a brain metaphysics, the concept would be very inadequate to its task if this *were* its actual task. Here, we should have a true application of Bacon's quip that anyone who supposed he could grasp reality through conceptual thought seemed to him like a man who, to gain better knowledge of a distant object, climbed a high tower and looked out from it even though he was perfectly free to approach the object itself and observe it close at hand. Here one thing is seen correctly: that the concept, in accordance with its characteristic attitude must, unlike direct perception, move its object off into a kind of ideal distance, in order to bring it within its horizon. The concept must annul "presence" in order to arrive at "representation." But for us this transformation no longer has the purely negative significance that it must have for strict positivism. The analysis of both perception and intuitive knowledge has shown us that even here this transition is required and within certain limits effected. The function of the concept does not create a break in the totality of knowledge—it continues a basic trend which already proved to be at work in the first stages of sensory, perceptual knowledge. And it is precisely in this continuation that the trend is truly authenticated and justified.

It has been argued that my critique of the theory of abstraction applies if one starts from the most highly developed concepts, those of mathematics and mathematical physics, but that it falls down as soon as we consider the prior stages of scientific knowledge, as soon as we take as our basis those concept formations which are already found—far from the aims of science—in our natural world view that is not yet modified by theories. Here, it is maintained, the theory of abstraction remains in full force, for the "intuitive concept" is actually developed from the "general mnemonic image" that has been deposited in us by a series of concrete sense perceptions. This attempt to rescue the abstraction theory was undertaken by Max Brod and Felix Weltsch in their book *An-*

*schauung und Begriff*. But it seems to me that precisely through the sharpness and pregnance with which it elaborates the essential features of the "abstractive" view of the concept, this work only reveals all the more clearly the dialectic in which the view must always ultimately become entangled. For according to this view, the true and essential achievement of the concept is to transform the sharp, individually determined images provided by sensation and perception into unclear and blurred ideas. This vagueness is regarded as the necessary condition for the concept—the element in which it alone can live and breathe. By detailed psychological analyses Brod and Weltsch seek to show how the perception and the intuitive image gradually enter into this element. Memory functions as a medium, for here begins that blurring of boundaries between the particular sense impressions that is taken up and continued by the concept.

Actually self-observation shows how rare separate remembered images are, i.e. images in which the memory of a unique, truly punctual experience has remained securely free from the influence of similar ensuing experiences. Almost always a remembered image represents a whole series of impressions. The memory of a friend represents him to me in many connections at once. If I think of a landscape, it stands before me as I have seen it time and time again, in varying extension, illumination, mood. But these images, which represent so much deviation, do not for that reason cease to be intuitive. Thus the general remembered image actually meets our . . . condition: to save the world from its infinitely progressing pulverization; ideas thus arise which bring back to a higher unity what is crumbling into disparate, detailed images. This mission is accomplished by the general memory image: as a blurred idea, which because it can be interpreted into many sharp, deviating images, embraces these images in itself. . . . In the alternately sharp and blurred parts of this general memory image there is a kind of copy of all the images we have experienced; they are all represented through the special stratification of the vagueness in the memory image.

By way of characterizing this relationship, Brod and Weltsch introduce a special symbol:  $A + x$ .  $A$  signifies what was common to the various experienced images—that is, for example, to the landscape in different illuminations and moods, whereas the divergent blurs into  $x$ . "In the

blur we have thus found the instrument by which two seemingly opposed attributes, hitherto regarded as crass contradictions, are brought into one: the intuitive and the abstract. For some intuitive images are at the same time abstract; these are the blurred images of the form  $(A + x)$ ." And here, it is claimed, we have for the first time a foundation for a true psychology of thought—insofar as we do not arbitrarily restrict thought to the realm of scientific knowledge but seek to apprehend it in the totality of its living manifestations. It consists then in nothing other than "the living play of the  $(A + x)$  formations": "It seems certain that we think in blurred general intuitions."<sup>27</sup>

With this, however, the Gordian knot of the problem of the concept is not untied but hacked to pieces. For are we really saved from the infinite diversity and fragmentation of individual impressions by fleeing from them into a blurred general idea? Can we, and do we, wish to renounce this multiplicity? Does not the meaning of concept formation consist precisely in the fact that it gives us an Ariadne's thread *within* the labyrinth of the many and the particular? The genuine concept turns away from the world of intuition only in order to lead back to it with all the greater certainty: it serves to determine the particular itself. Here it cannot be argued that this function resides only in strictly scientific concepts. For although it is in the scientific concepts that this function of the concept first stands out with full pregnancy, although it is here most clearly apprehended and immediately accessible to logical analysis, still it is not limited to the scientific concepts. It already belongs to those preliminary stages, those germ cells of the theoretical-scientific concept which Brod and Weltsch designate as "intuitive concepts." For these too are not so much generic concepts as concepts of combination. They do not present blurred general images of things but strike bridges between the things that are given in perception as merely singular and relatively isolated. Thus for example the intuitive image of color is no generic image in which red and blue, yellow and green fade into each other in some vague way; through the image of color, rather, a characteristic field is marked off from sensory experience as a whole and "defined" by a definite factor of relation, the relation to light and to the eye. How would such an insight into the order, the articulation, the concrete differences of a multiplicity be possible if the concept consisted essentially in a turning

27. Max Brod and Felix Weltsch, *Anschauung und Begriff. Grundzüge eines Systems der Begriffsbildung* (Leipzig, 1913), pp. 72 ff., 144.

away from them, a leveling of these differences? <sup>28</sup> And is it not a leveling when, understanding the differences through the concept and deriving them from it, we suppose them rather to be blurred in it?

But if instead of merely examining these conflicting views of the concept, of the concept as such, we inquire into the underlying reasons for them, we find ourselves once again carried back to our central problem, the problem of representation. For it is the view of representation and the "conditions of its possibility" that dominates and determines the view of the concept. If Brod and Weltsch have recourse to the "blurred image," it is explicitly because in their eyes only such an image, only an image that is not thoroughly determined but is in a sense opalescent, possesses the power of representing a multiplicity of contents. This relative indeterminacy of an image seems to be the sole basis for its having meaning, seems alone to give it the possibility of being taken now in this and now in that sense. "The property which blurredness has of meaning something," it is thus concluded,

gives the ( $A + x$ ) in germ that primary characteristic of the concept, which has created so much difficulty for theoreticians, namely its extension side by side with its content. . . . How must a single image be constituted in order to serve as a denominator for many objects? On the basis of all the foregoing we may answer as follows: An ( $A + x$ ) with its ability, within the limits which its  $A$  imposes on its  $x$ , to transform itself into different images and thus link itself without difficulty with these recent, disparate images through a judgment of identity, can name the objects corresponding to these images. The

28. Cf. my *Substanzbegriff und Funktionsbegriff*, pp. 23 ff. In agreement with the view there put forward, Burkamp has recently written: "From individual things we rise to concepts such as 'chair' and 'dog,' and then to still higher concepts such as 'living creature,' 'body,' 'mass.' From individual states we rise to concepts of 'quantity of electricity,' 'strength of current,' 'energy.' From the individual numbers we rise to the concepts of the prime number and of 'number' in general. Between these concepts we set the connecting law. . . . But this lawgiving acquires meaning only through the fact that we can return downward to the basic stages. The law that applies to body and mass will now, on the strength of the law of the logical concept, also be valid for 'chair' and 'carpet,' and finally also for the individual chair that may lie in my path. This individual chair is now enriched in its being for me by the interweaving of all the concepts under which it stands. . . . In all this the enrichment of the individual is grounded in the knowledge of the general, of the laws that apply to the general concept. This enrichment in the knowledge of the particular and especially of the individual is the purpose of the whole hierarchy of concepts. . . . It is for the sake of the lower levels that we work in the higher levels" (*Begriff und Beziehung*, first study, pp. 2 f.).



power of the  $(A + x)$  to be the subject of different identity judgments makes possible the concept's function of denomination. Thus two distinctly disparate individual images, the image of the dog lying ( $L$ ) and that of the dog standing ( $S$ ), may be given to me. But if from  $L$  and  $S$  and the other positions of a certain dog which are known to me now I form its  $(A + x)$ , i.e. if I detach from them a blurred general image of "the" dog, the image of lying ( $x_1$ ) or that of standing ( $x_2$ ) may be added to this image, and accordingly the  $(A + x)$  can designate for me now an  $(A + x_1)$  and now an  $(A + x_2)$ .<sup>29</sup>

But if we look back over our previous reflections, it is this context that best clarifies the contrast between our view of representation and the view that is here expressed. For at every step we have had to combat this very assumption that the symbolic meaning of an image, what lends it a definite significance, may be disclosed as something in itself, as a real, distinguishable part of it. "Meaning" and "existence" are not homogeneous in the sense that they may be disclosed as components of an image which they "compose." The very formula that is here chosen as the expression of the concept must appear questionable insofar as it joins the  $A$  and the  $x$ , the expressions of the universal and the particular, by a simple plus sign. Can the universal and the particular, the content and extension of a concept, what is "meant" in the concept and what is "given" by perception or sensory intuition, really be added in this way? Such an addition transforms the organic unity that characterizes and distinguishes the concept into an aggregative juxtaposition. In the propositional function  $\phi(x)$  that designates a certain concept, the expression for the function itself and the expression of the particular values that are grouped together by it do not stand on the same line: the "factors" that are here brought into relation with each other cannot be conceived as elements of a sum. There is an inherent contradiction in attempting to make the term  $\phi(x)$  intelligible by dissecting it into separately existing ingredients, by making the  $\phi(x)$  into a  $\phi + x$ . For the function sign  $\phi$  is not an expression for a single numerical quantity that might be combined with other quantities of the variable by elementary arithmetical operations. We have above compared the "concept" with the "universal member" of a series, which designates the rule of the succession of its individual mem-

29. Brod and Weltsch, pp. 77 ff.

bers. This law of the series restricts the individual elements belonging to it to definite conditions; but it does not itself constitute a member of the series. If an arithmetical series of the form  $\frac{1}{2}$   $\frac{2}{3}$   $\frac{3}{4}$   $\frac{4}{5}$  etc. is designated by the formula  $\frac{n}{n+1}$ , this  $\frac{u}{n+1}$  no longer designates an individual magnitude; it stands rather for the whole of the series, insofar as this series is taken not as a mere sum of parts but as a characteristic relational structure. Similarly, to choose a geometrical example—the universal concept of the conic section is not gained through images of individual circles, ellipses, parabolas and hyperbolas flowing together and forming a blurred general image; what happens rather is that circle and ellipse, hyperbola and parabola are retained as thoroughly determined geometrical forms, but at the same time are moved into a new relational context; they all obtain the trend and characteristic vision toward the right cone, from which they may arise as results of the various sections that may be made in it. And the same is true in principle of the simplest intuitive concepts. They never form a mere conglomerate of sense impressions and remembered images but contain a peculiar articulation of these impressions and images, a form of organization. In them the separate is “seen together”—not in the sense that its components are mingled but in the sense that their connection in regard to some linking factor is retained. When the Greek language designates the moon as the “measurer” ( $\mu\eta\nu$ ), and the Latin language as the “glittering” (*luna*), different intuitive concepts underlie these different designations—but these concepts act in both cases only as a factor of comparison and correlation, as a point of view, which is not itself given as either a distinct or a blurred visible thing. And here it is largely immaterial whether this point of view asserts itself in the further objective progress of knowledge or is superseded by another mode of vision. Such changes mark the content and scientific validity of the concept, but not its sheer form. If, for example, certain languages designate the butterfly as a bird, the connection thus expressed must of course be severed as soon as thought progresses to describing the zoological orders systematically in accordance with definite scientific, morphological, or physiological criteria—but the original viewpoint of classification which focuses not on such criteria but solely on the intuitive factor of “flying” is not thereby declared to be absolutely meaningless, but merely to represent a standard of meaning which from the standpoint of scientific synopsis must be replaced by another, more

complete one. The circumstance that such a change of standard proves to be necessary in the transition from intuitive to scientific concepts does not prove that the operation of measurement as such is not already practiced in the prescientific concepts—that they, too, do not already follow determined rules of relational thinking. In the theory of Brod and Weltsch, however, at least the prescientific concept—for in connection with the scientific concept they restrict their thesis in very important and even crucial respects<sup>30</sup>—is produced by a mere flowing together of representations and remembered images. In this theory, consciousness resembles a photographic plate, on which in the course of time various images are produced which overlap and mix with one another, until they finally become a single unclear general image.<sup>31</sup> But even if we accept this metaphor as an expression for the genetic process of concept formation, it remains impossible to see how it can elucidate the logical function of the concept, its ability to “name” and designate various particular intuitions. For its origin in the particular impressions can never in itself enable the concept to *represent* exactly that from which it sprang. Admitted that a general image of this sort is formed on the photographic plate; still, the plate will never be able to know it as such, to refer it back to the particular elements from which it grew. Such a relation would require the process in which the concept was acquired to be in a sense annulled, and the elements from which it is composed to be freed from the mixture into which they have entered. If we attribute to the photographic plate the faculty of mixing all the particular impressions that are made on it, shall we also impute to it the power of separating them? Yet precisely this is presupposed and required in “representation” in the strict sense. Every function of representation embraces within it an act of differentiation—and both must be conceived not as a mere succession but as a genuine involvement—the positing of identity must be performed in the differentiation and vice versa. For this kind of systole and diastole, or syncrisis and diacrisis, of concepts, all analogies drawn from the world of things and from its processes are inadequate. Here only the opposite formulation of the problem carries us farther; here we must begin with what the concept *means*, and proceed to what it proves to be in objective knowledge and what it performs for the building of this

30. Cf. esp. the critical argument against my *Substanzbegriff und Funktionsbegriff* in Brod and Weltsch, pp. 234 ff.

31. Brod and Weltsch, pp. 74 f.

knowledge. Conversely, we can never understand the basic spiritual act of "representation," of intending a "universal" in the particular, by dissecting it and in a sense smashing it into bits. When we do this, we do not retain the fragments, the parts of representation; rather, we pass altogether from the area of their meaning to an empty existence, whence no road leads back to the sphere of meaning.<sup>32</sup>

32. For a more complete treatment of this context I refer the reader to my article "Erkenntnistheorie nebst den Grenzfragen der Logik und Denkpsychologie," *Jahrbücher der Philosophie*, 3 (1927), 55 ff.