Conquering the Busy Beaver

presented by Kyle Ross

4th December 2002

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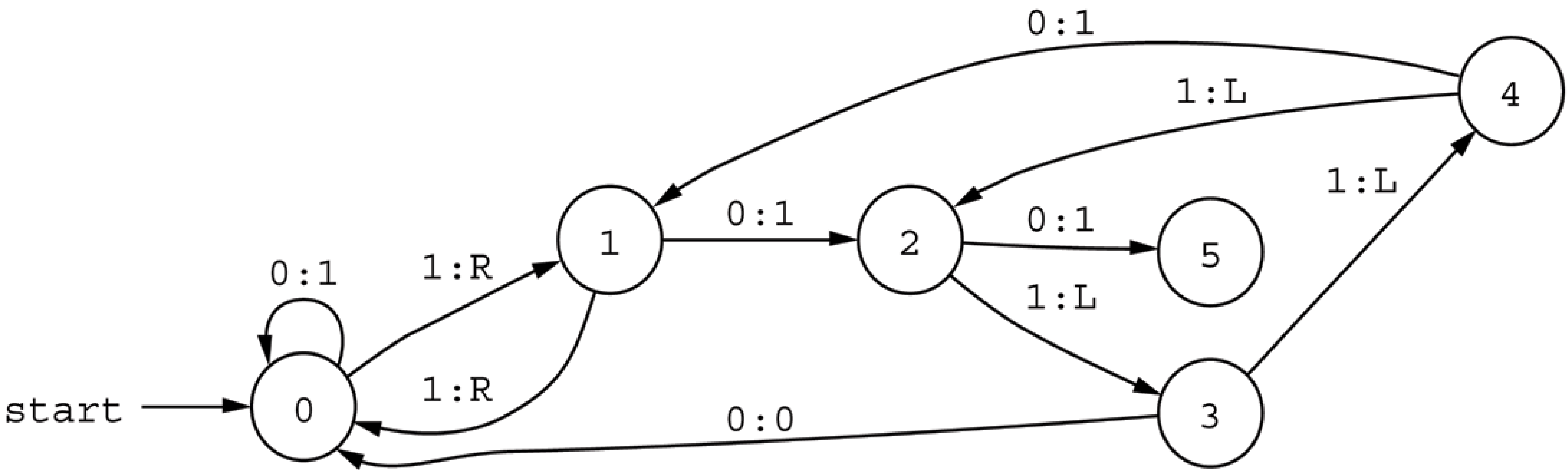
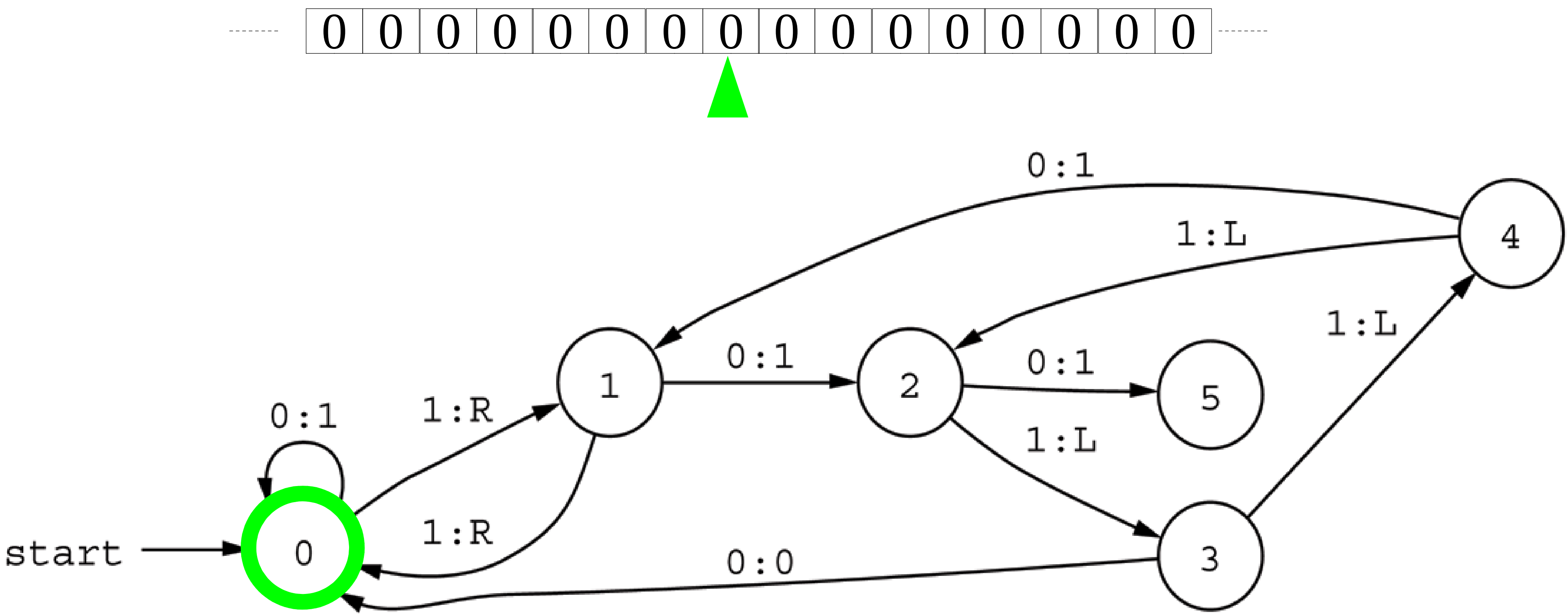
Turing Machines

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0



Gregg's Challenge

Turing Machines



Gregg's Challenge

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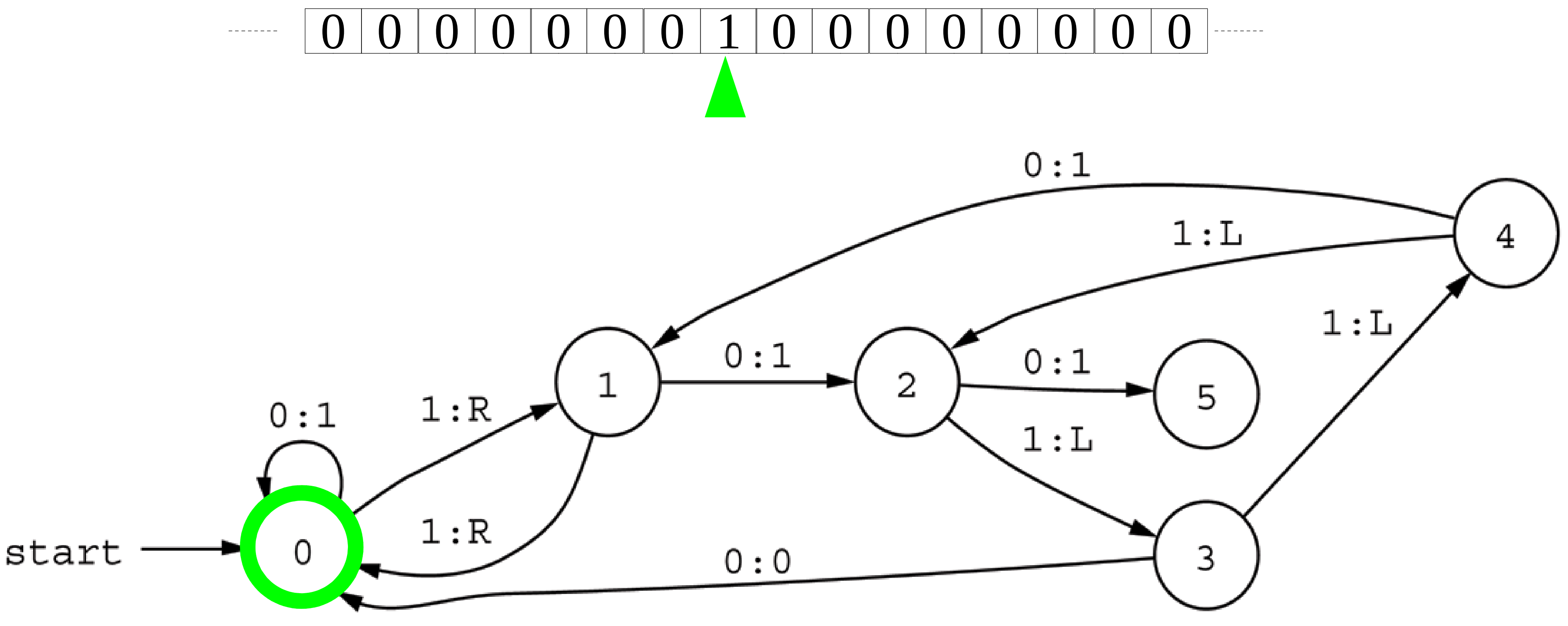
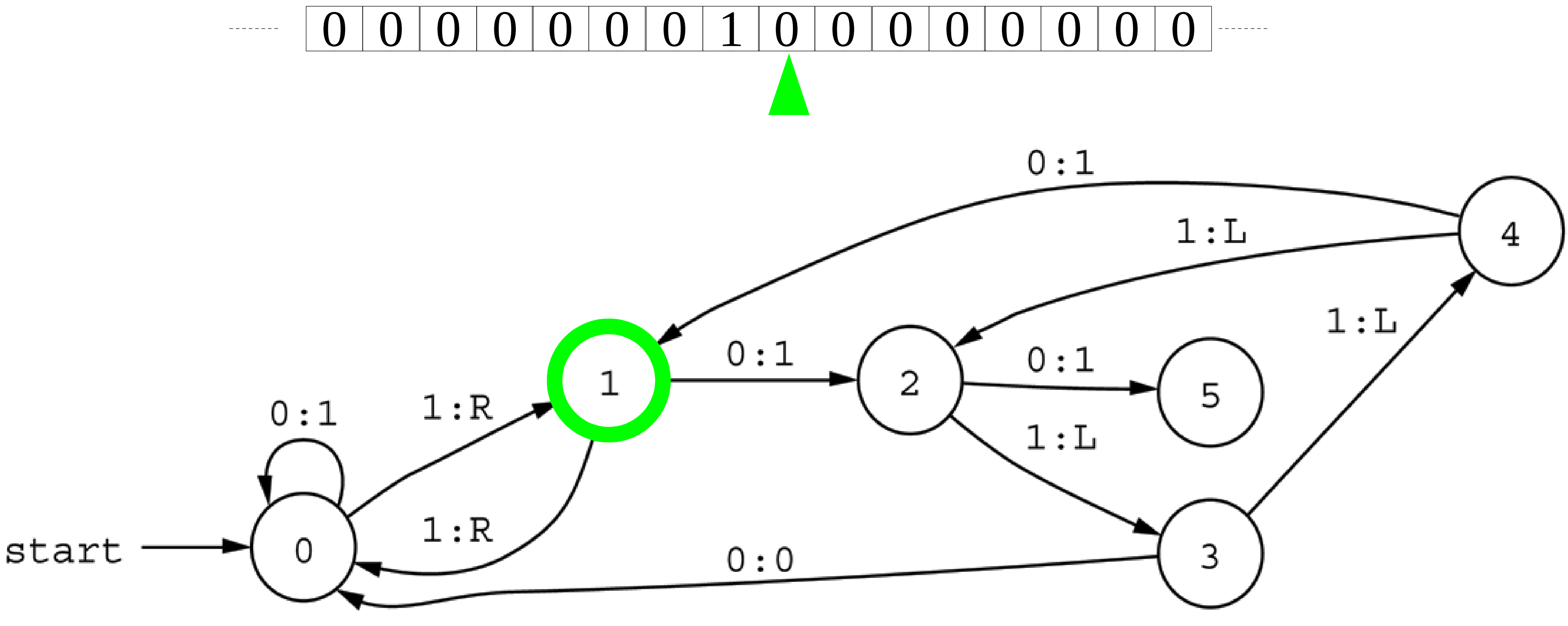


Gregg's Challenge

Turing Machines



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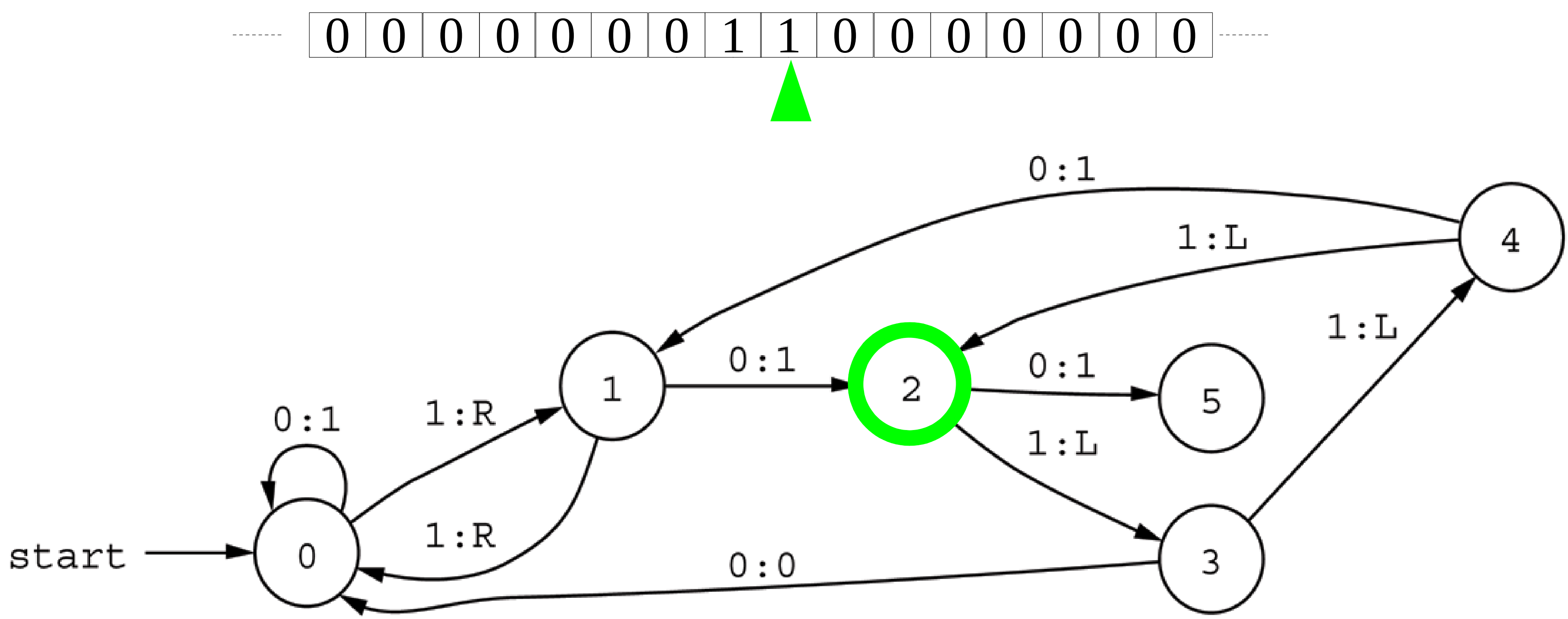
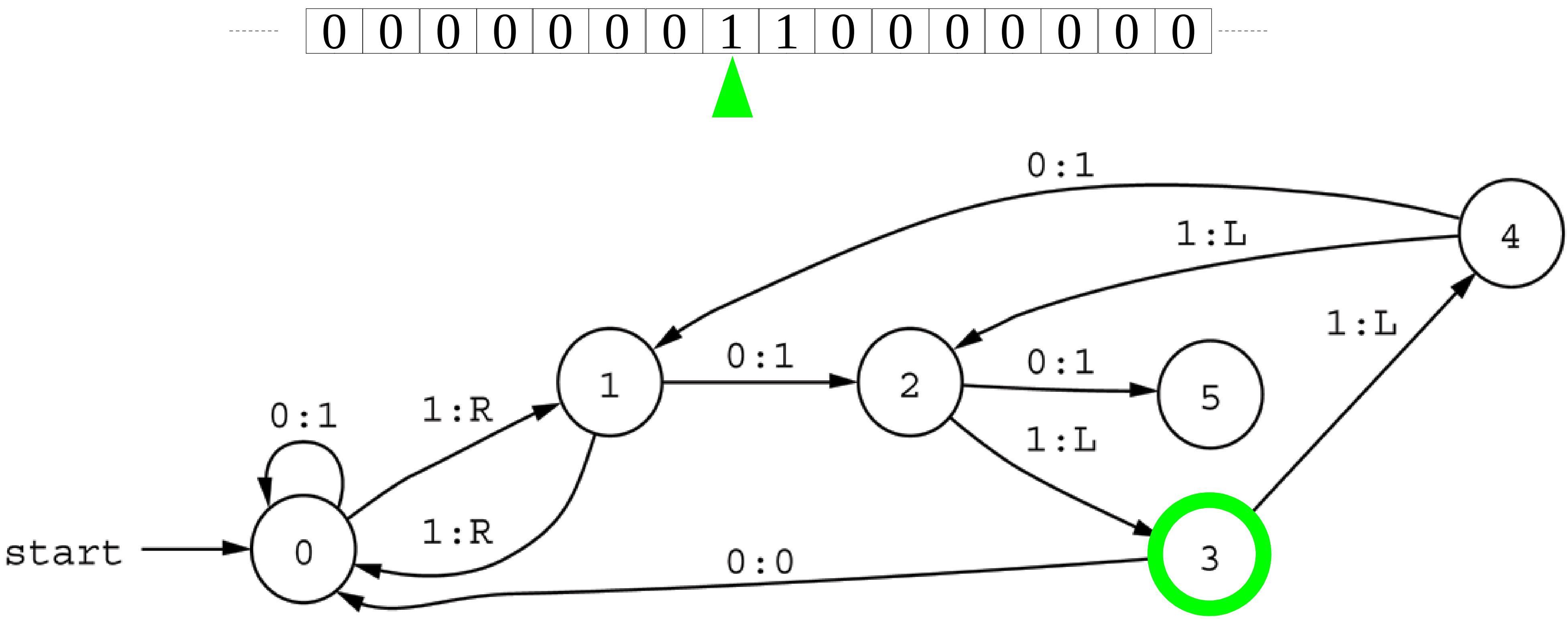


Gregg's Challenge

Turing Machines



Gregg's Challenge

The Busy Beaver Problem

"Consider, for a fixed positive integer *n*, the class K_n of all the *n*-card [state] binary turing machines ... Let *M* be a Turing machine in this class K_n. Start *M*, with its card 1, on an all-0 tape. If *M* stops after a while, then *M* is termed a *valid entry* in the *BB-n* contest ... and its score $\sigma(M)$ is the number of 1's remaining on the tape at the time it stops ... [the set of σ-values] has a (unique) largest element which we denote by $\Sigma(n)$ ... It is practically trivial that this function $\Sigma(n)$ is not general recursive ... [but] it may be possible to determine the value of $\Sigma(n)$ for particular values of *n*."

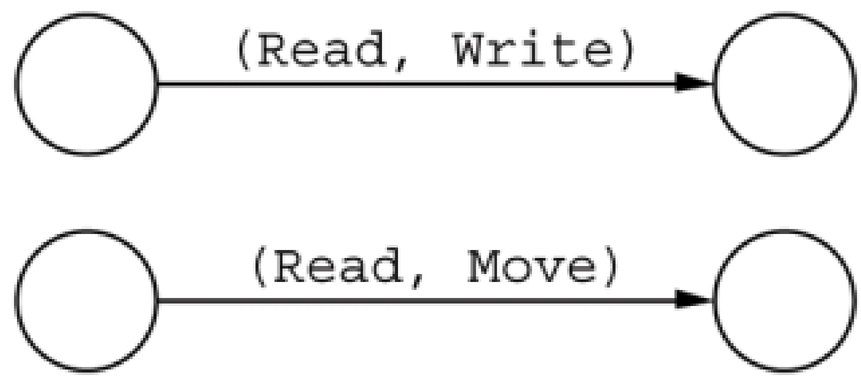
-Lin & Rado "Computer Studies of Turing Machine Problems" Journal of the Association for Computing Machinery, Vol. 12, No. 2 (April, 1964), pp. 196-212

Variants of the Problem

- quadruple vs. quintuple
- standard position vs. arbitrary format output
- implicit vs. explicit halt machine

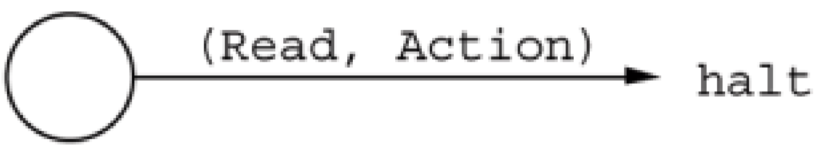
Turing Machine Formulations

quintuple formulation



quadruple formulation

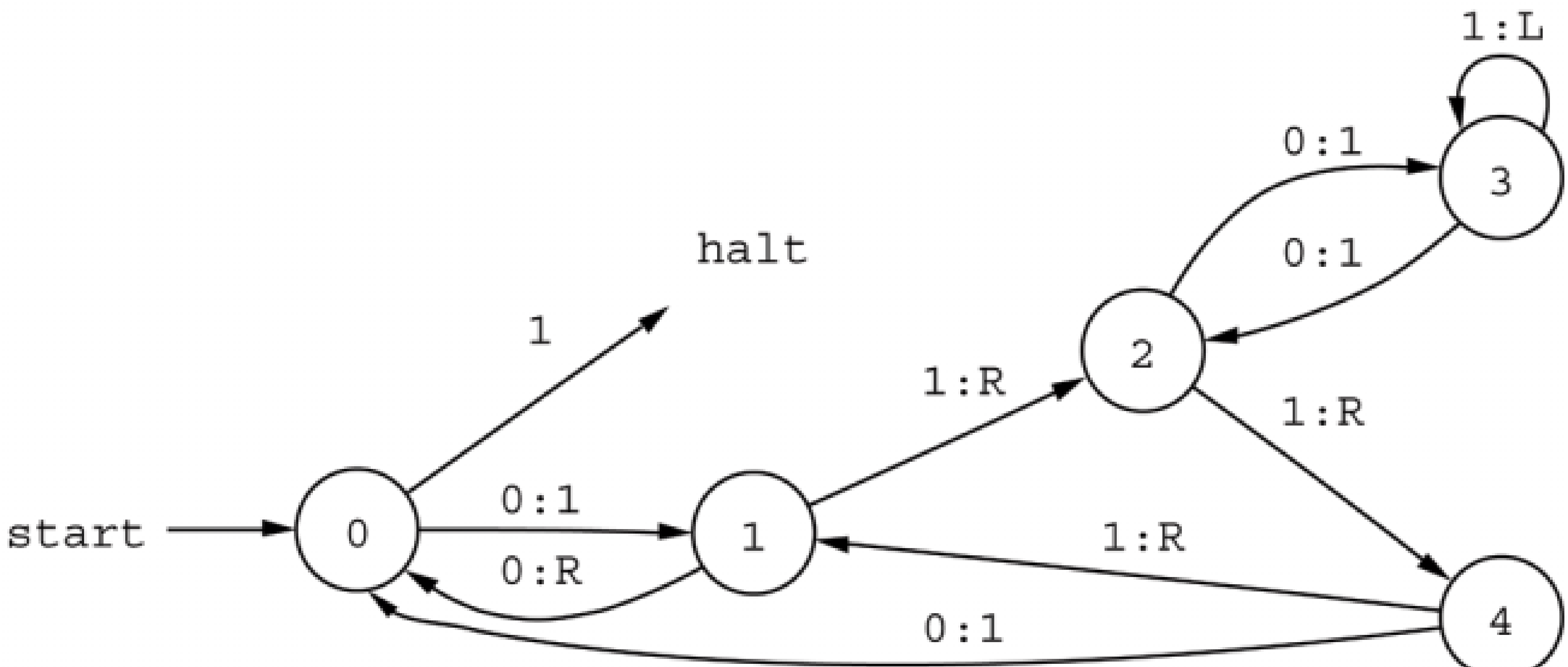
Turing Machine Formulations



explicit halt

implicit halt

Turing Machine Formulations



B(5)-11

Previous Work on Quadruple

- $R(n)$ - quadruple, explicit, no restriction
  - [nobody?]
- $O(n)$ - quadruple, implicit, no restriction
  - Oberschelp et al.
- $P(n)$ - quadruple, explicit, standard
  - Pereira et al.
- $B(n)$ - quadruple, implicit, standard
  - Boolos and Jeffrey

Known Results

n	R(n)	O(n)	P(n)	B(n)
1	***1***	1	1	1
2	***>=2***	2	2	2
3	***>=4***	3	4	3
4	***>=8***	8	***>=7***	***>=5***
5	***>=16***	15	***>=16***	***>=11***
6	***>=71***	***>=70***	***>=41***	>=25
7			>=164	>=164
8			>=384	

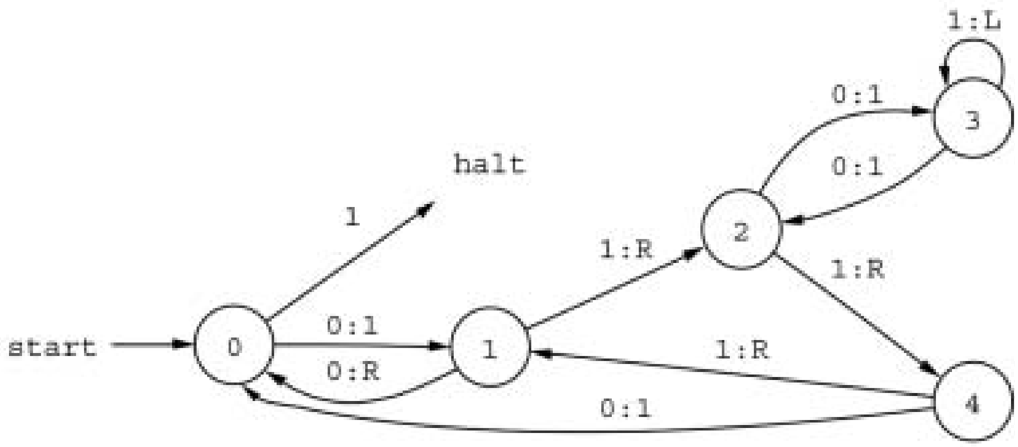
About the Quadruple Formulation

- Turing's World & Greg's challenge
- less-productive than quintuple machines
- greater room for optimisations

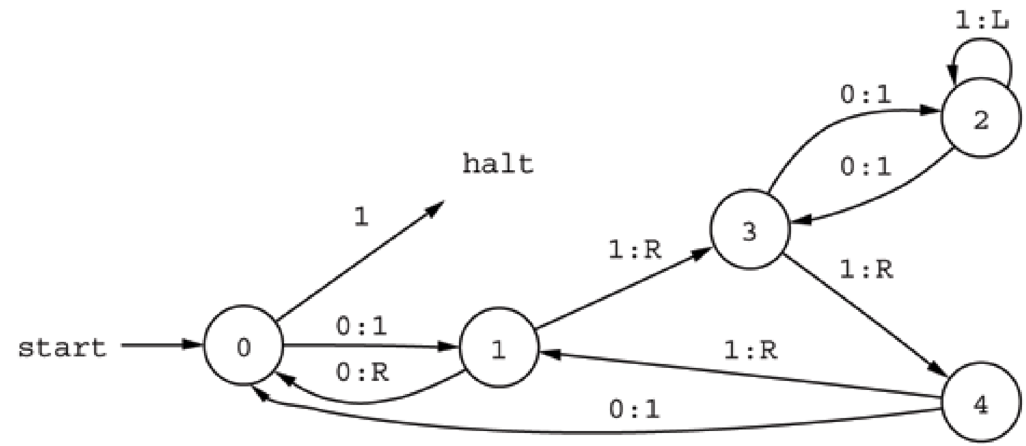
The Search Space

- $|M(n)|=(4n+1)^{2n}$
  - 4 possible actions for each of *n* next states
  - 1 no-action transition to halt-state
  - 2*n* possible transitions
- for $B(6)=(4(6)+1)^{2(6)}=5.96 \times 10^{16}$ machines
- not hopeless!

Inefficiency: Isomorphisms

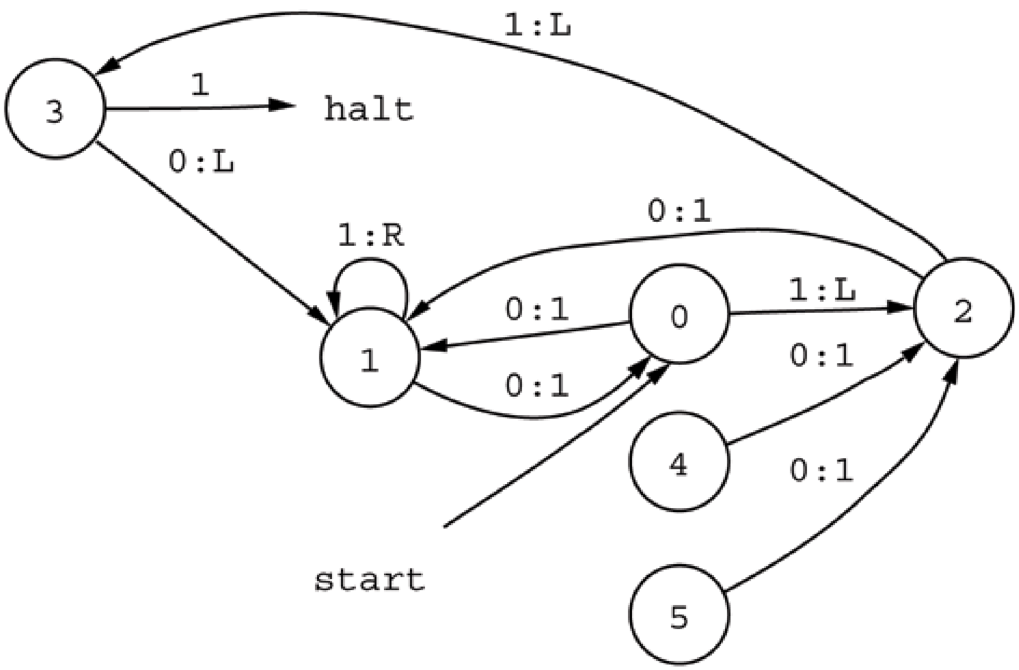


B(5)-11

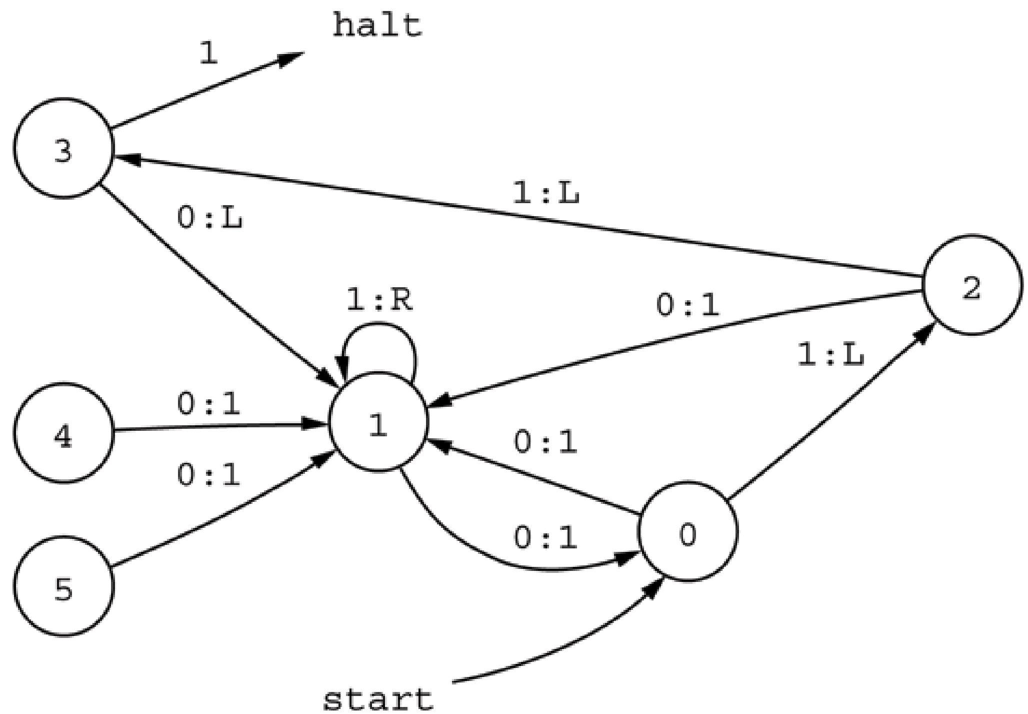


B(5)-11-isomorph

Inefficiency: Unused Transitions



B(4)-5-u1



B(4)-5-u2

Solution: Tree Normalisation

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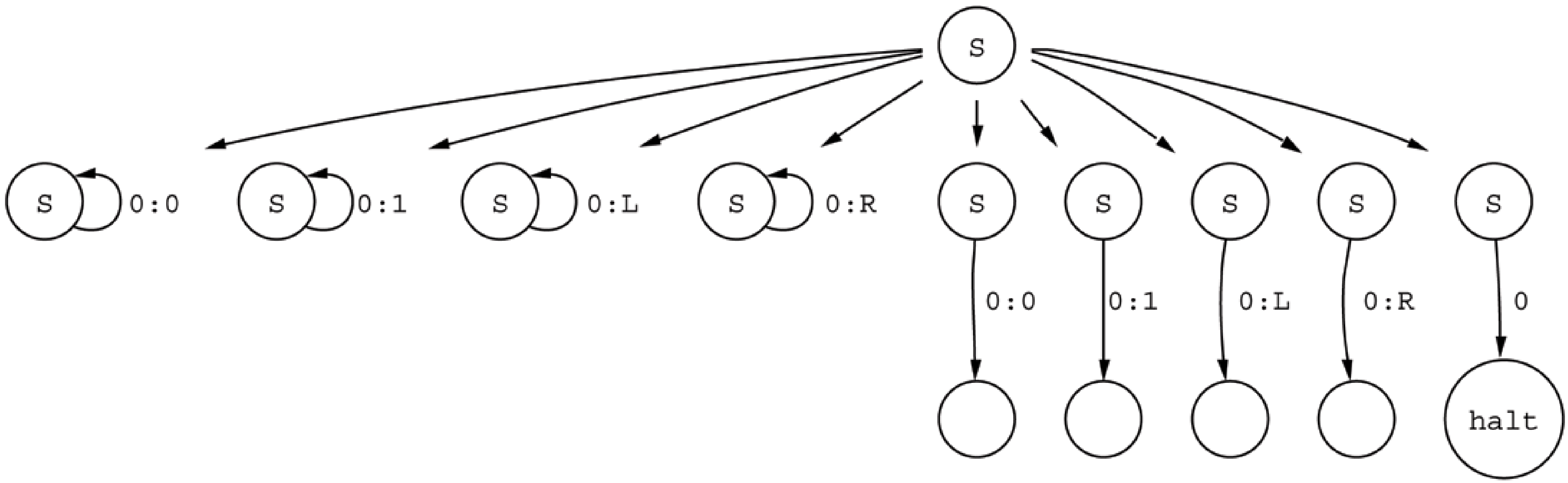
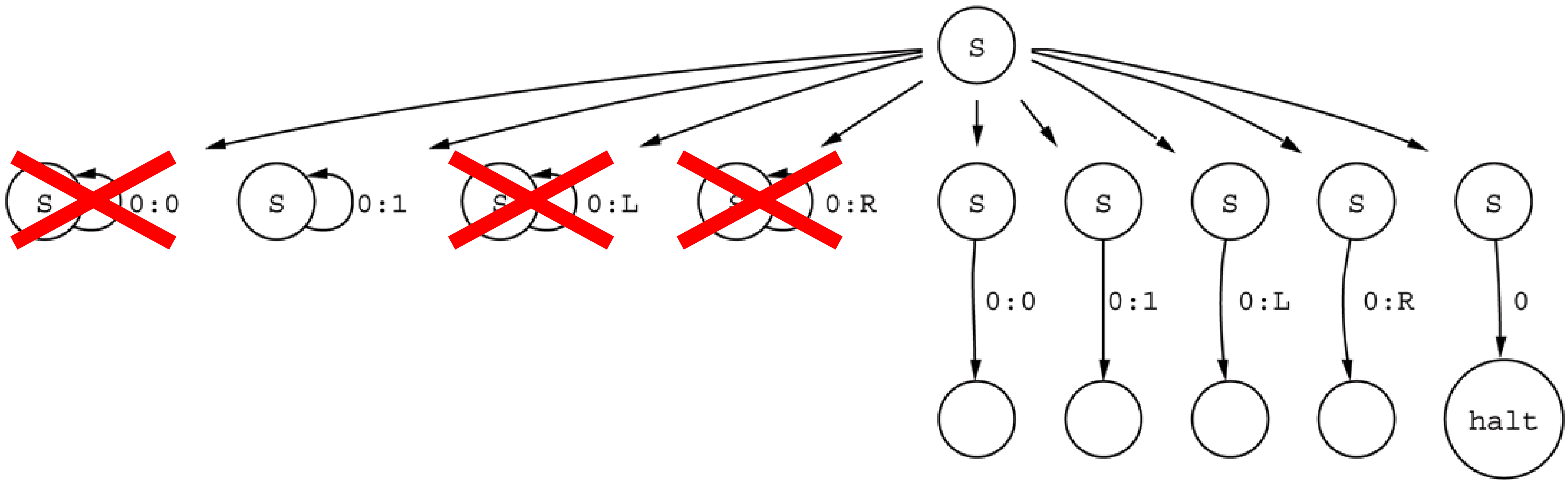
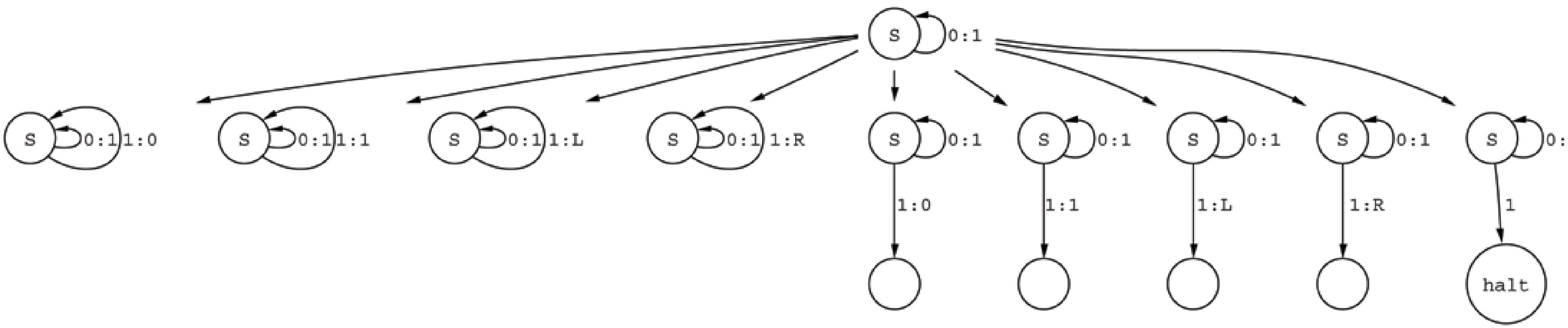


Solution: Tree Normalisation



~~non-halter~~

Solution: Tree Normalisation



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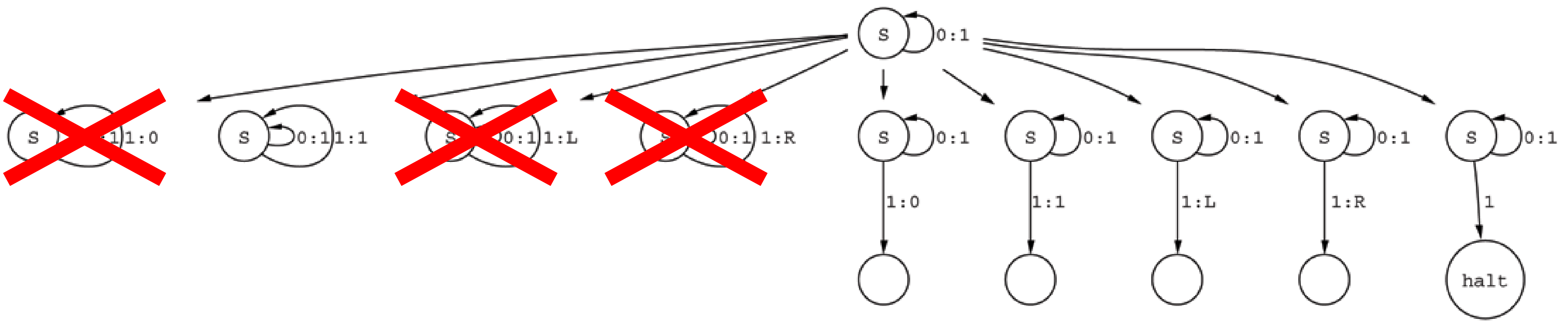
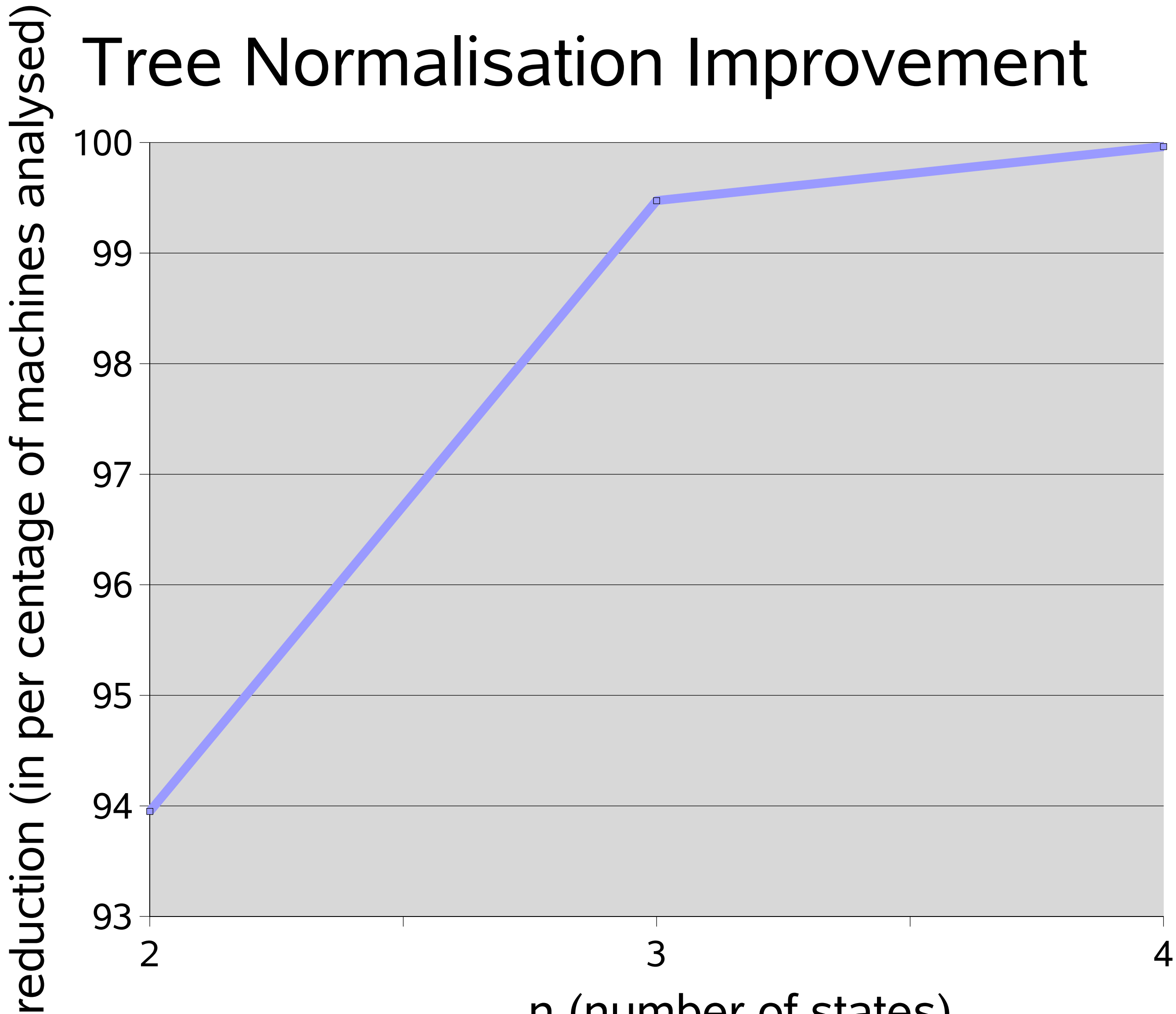


non-halter

Features of Tree Normalisation

- complete & optimal search
- no loss of absolute numbers
- great speed-up over pure brute-force

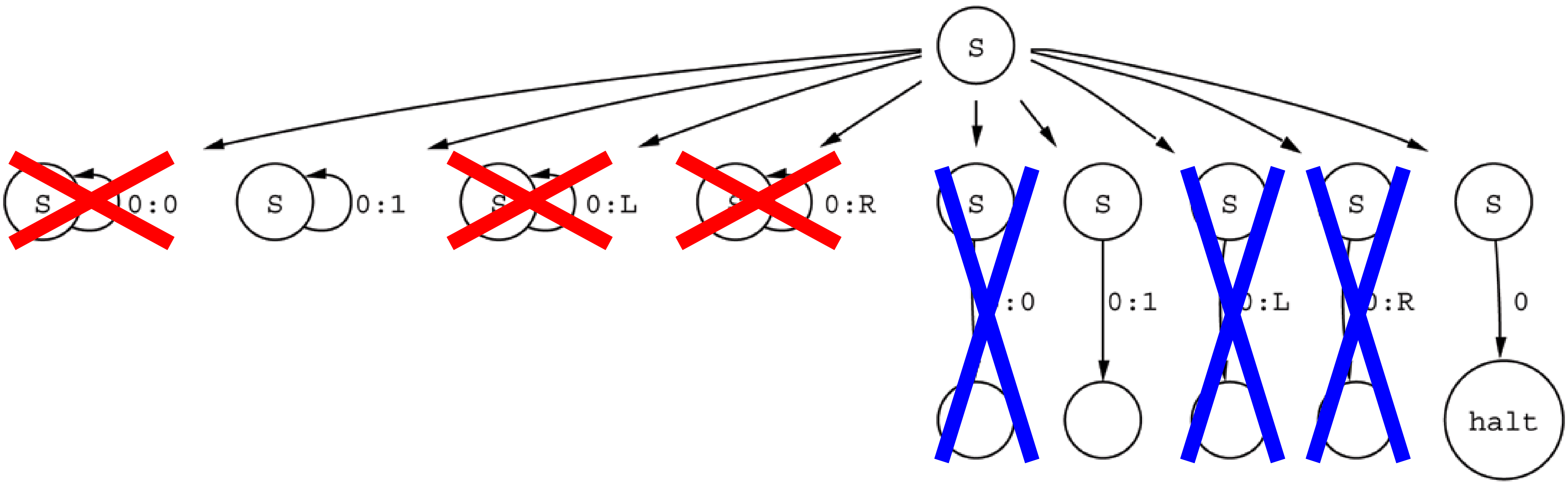
Improvement from Normalisation



Inefficiency: Empty Tape Machine

- machine reaches an empty tape after 1 or more shifts
- any machine that does not write 1 as its first action is such a machine

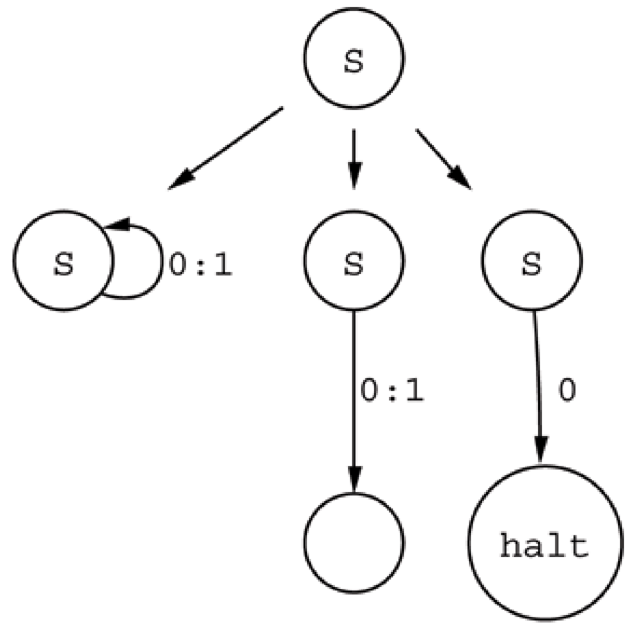
(Partial) Solution: Force First Write



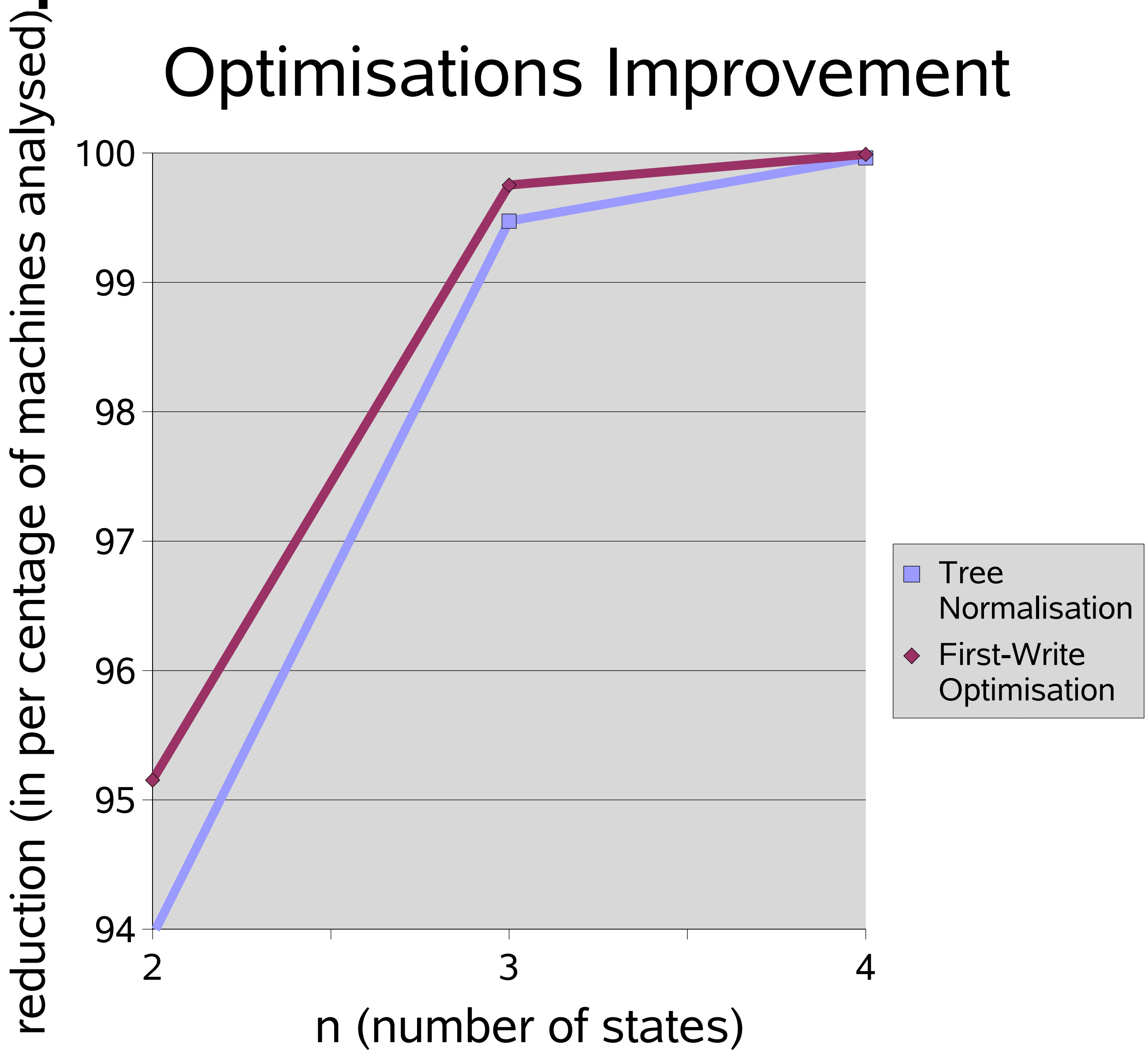
non-halter

first write not 1

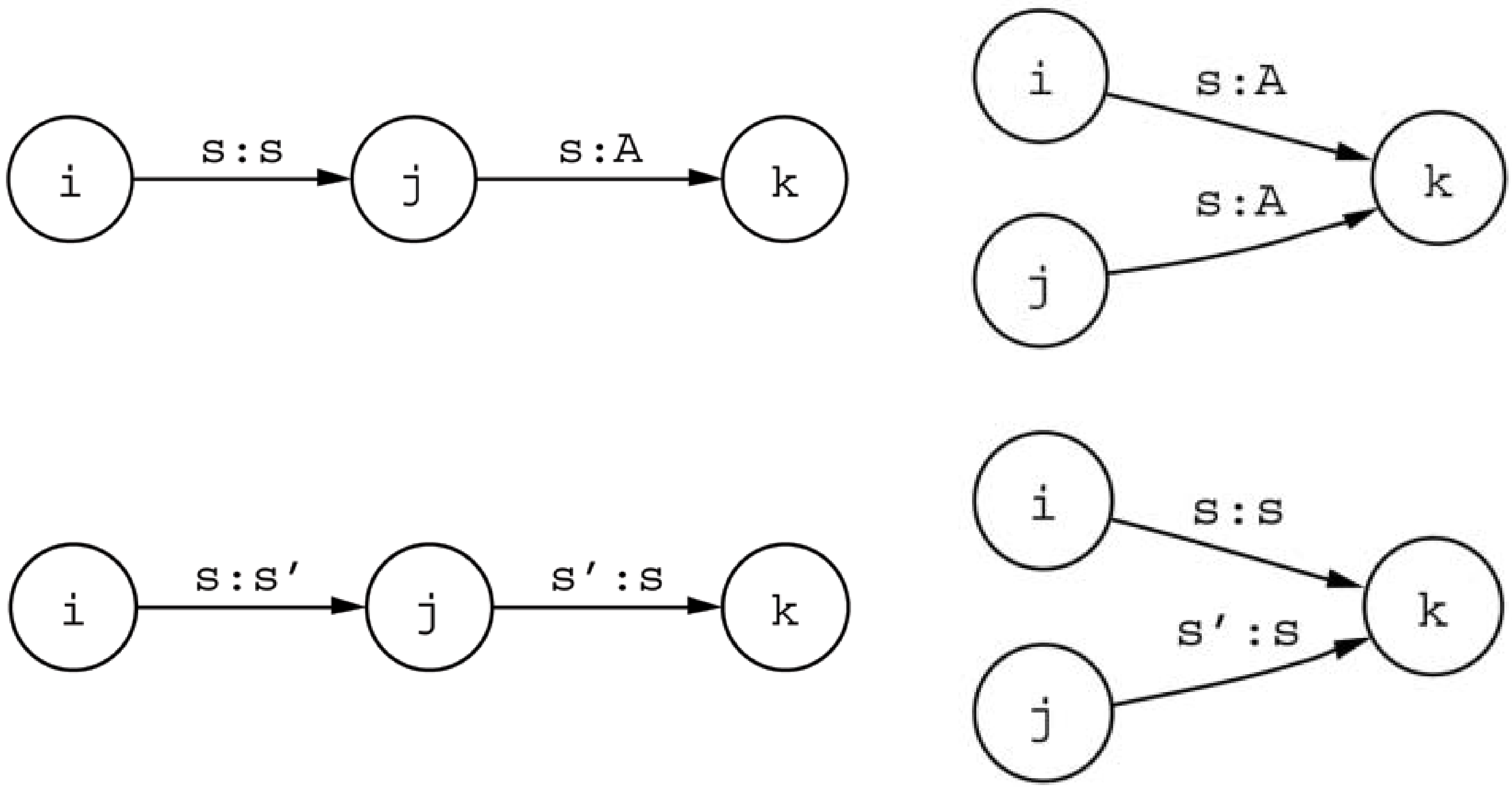
(Partial) Solution: Force First Write



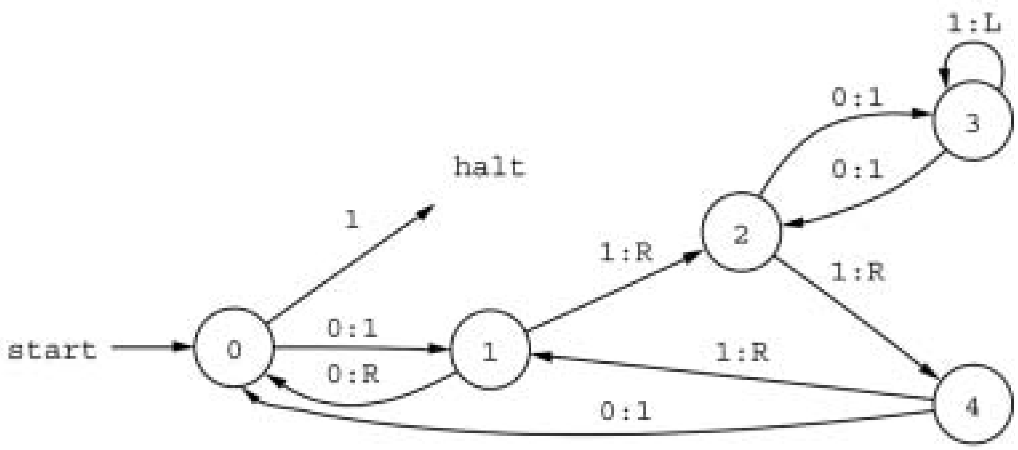
Improvement from First Write



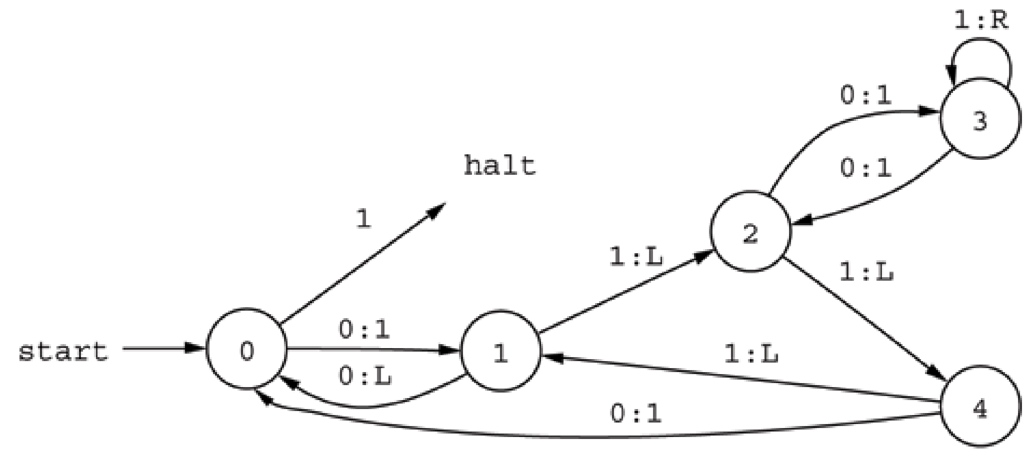
Inefficiency: Nonproductive Transitions



Inefficiency: Mirror Machines

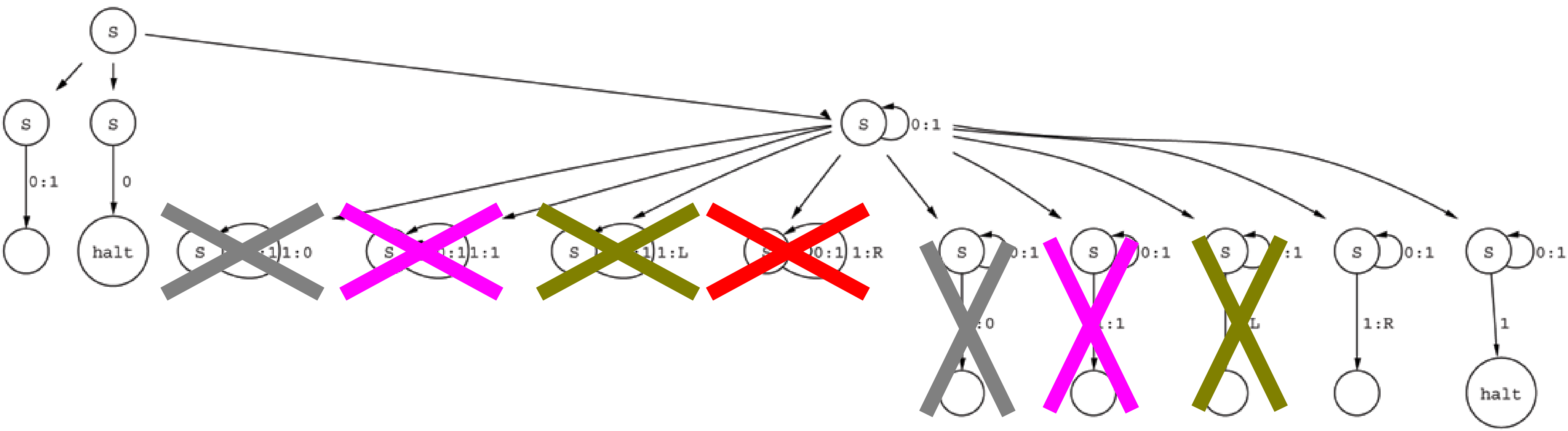


B(5)-11



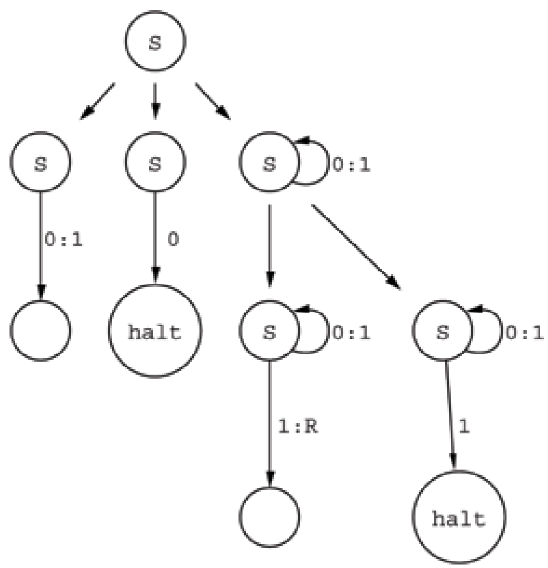
B(5)-11-mirror

Solution: Force First Move

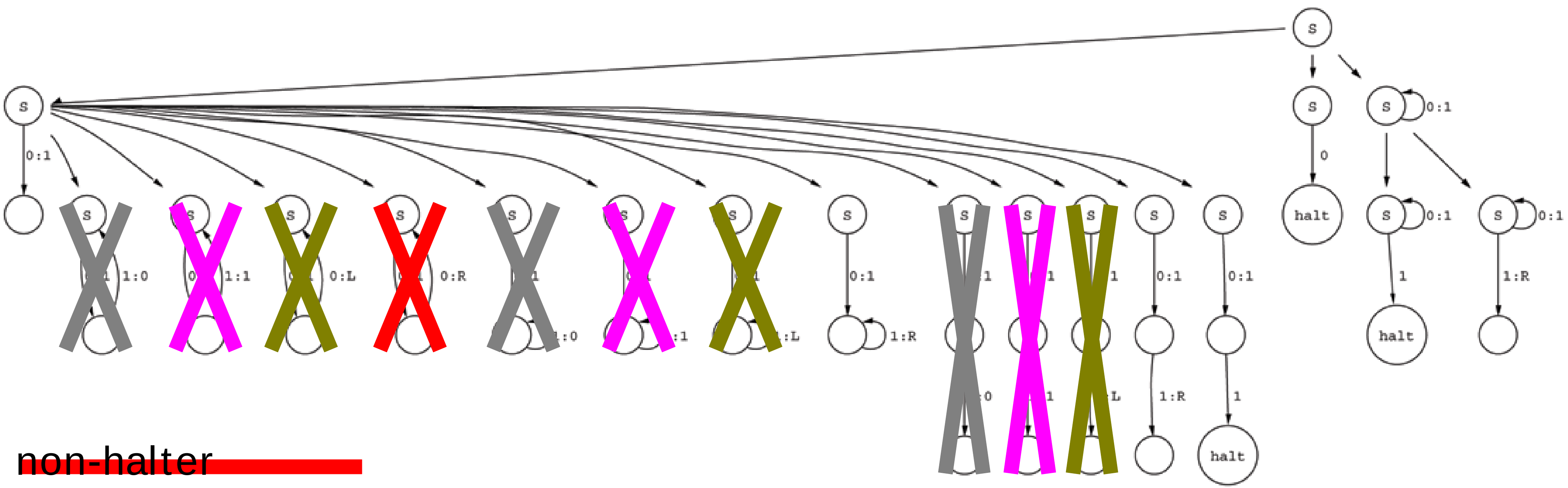


non-halter
first move not R
first write not 1
s:s transition
s:s'-s':s transition

Solution: Force First Move



Solution: Force First Move



~~non-halter~~

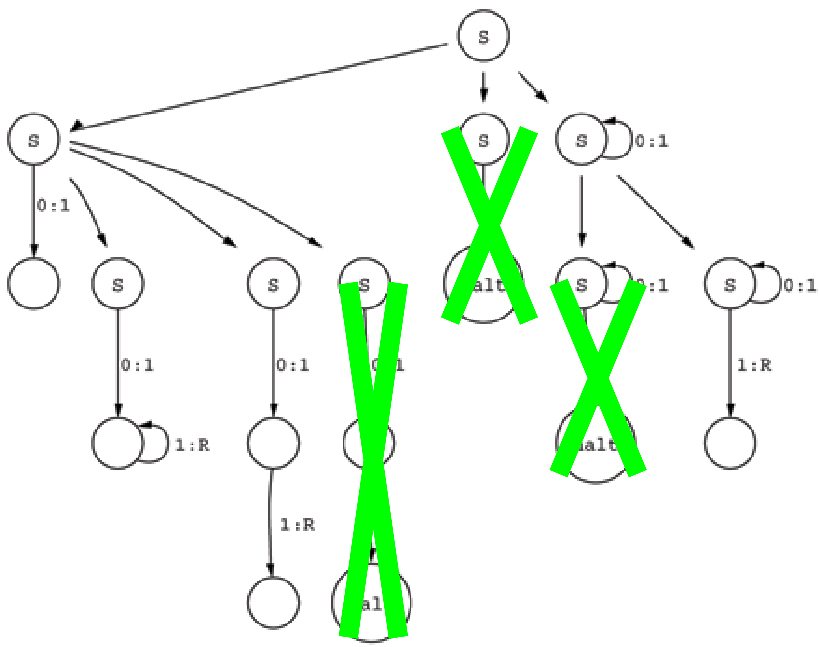
~~first move not R~~

~~first write not 1~~

~~s:s transition~~

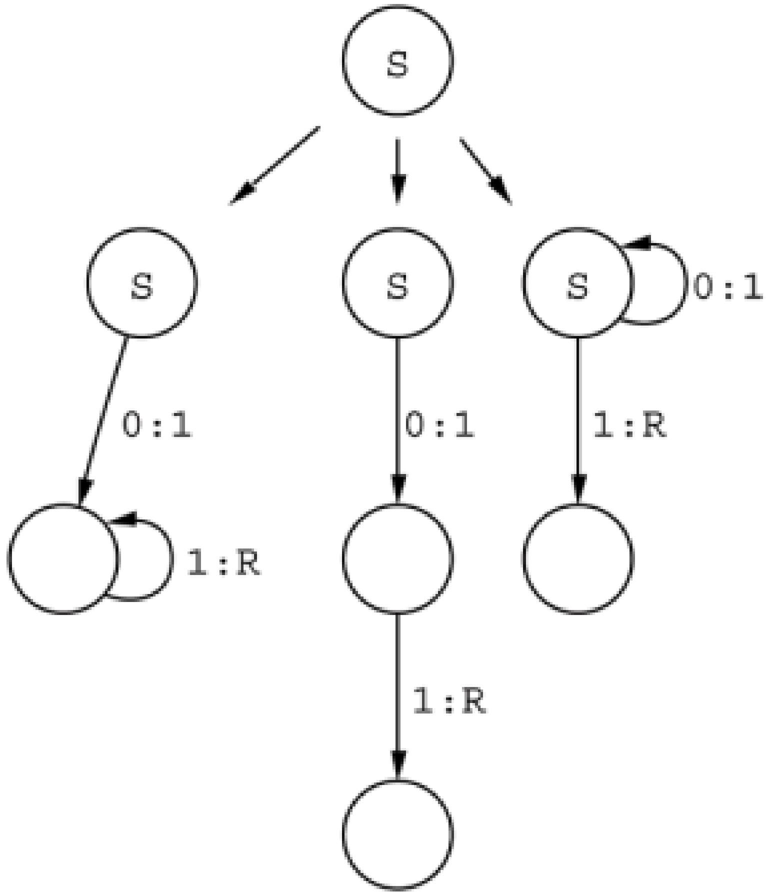
~~s:s'-s':s transition~~

Solution: Force First Move

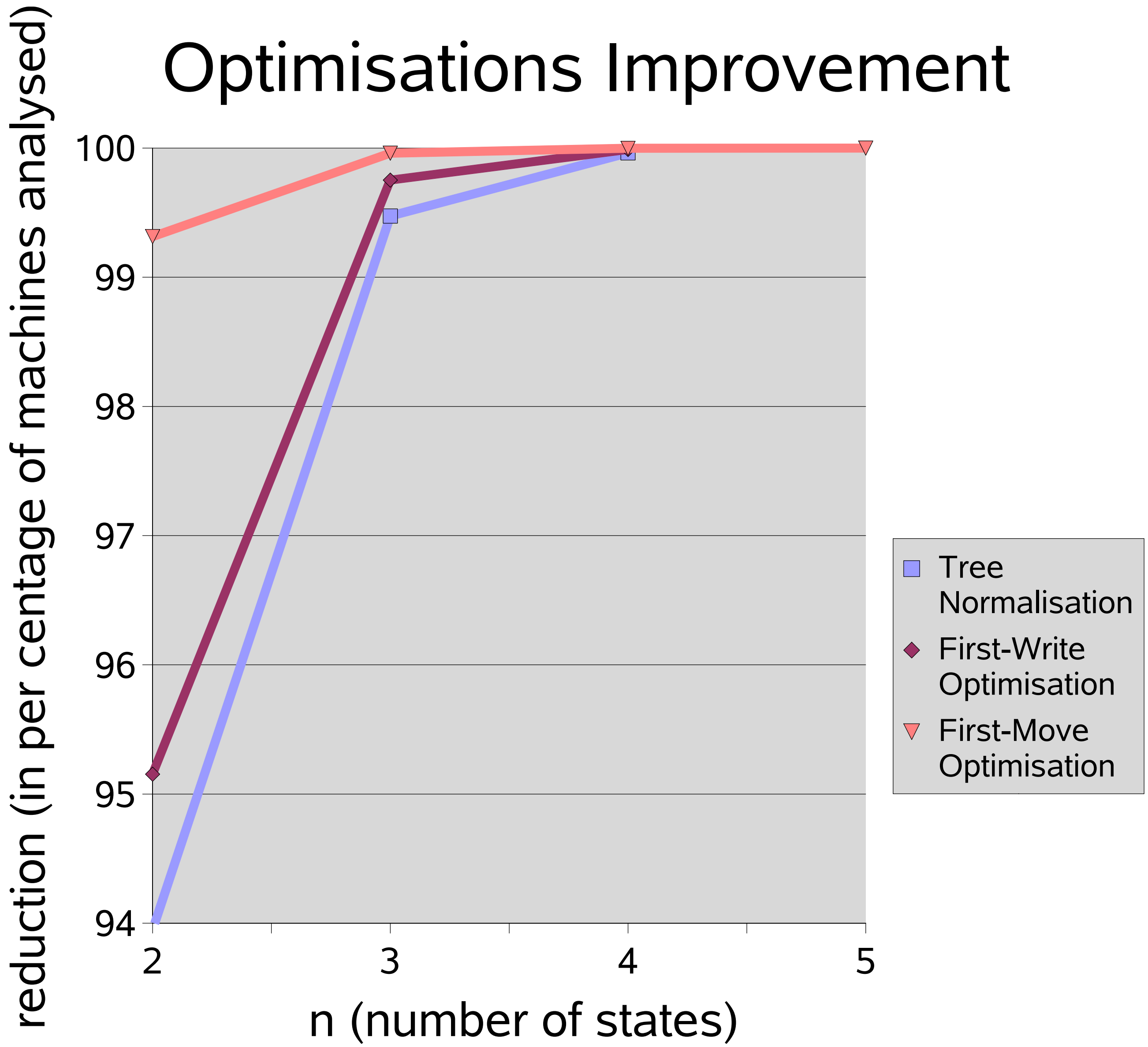


~~low-productivity~~

Solution: Force First Move



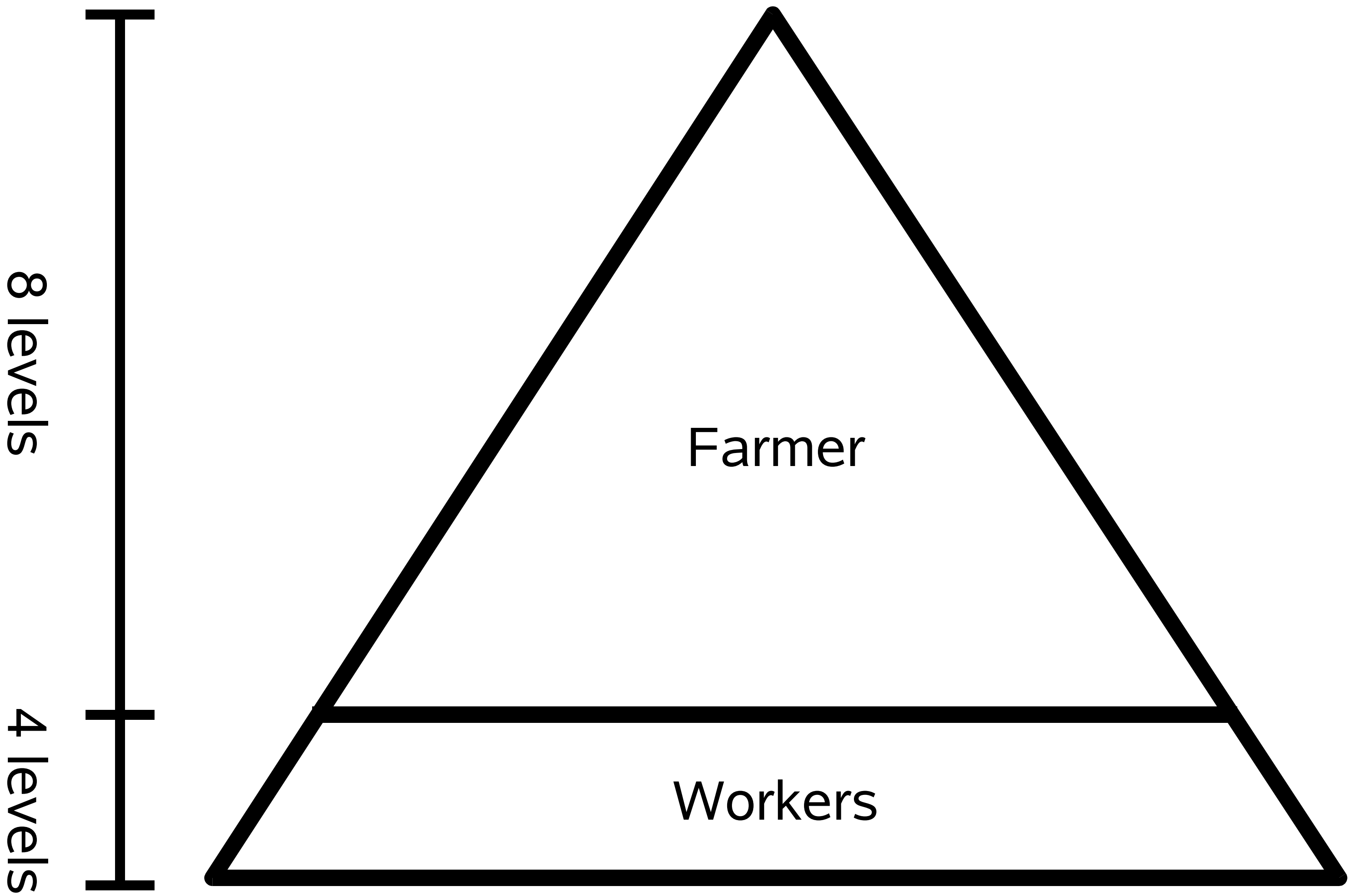
Improvement from First Move



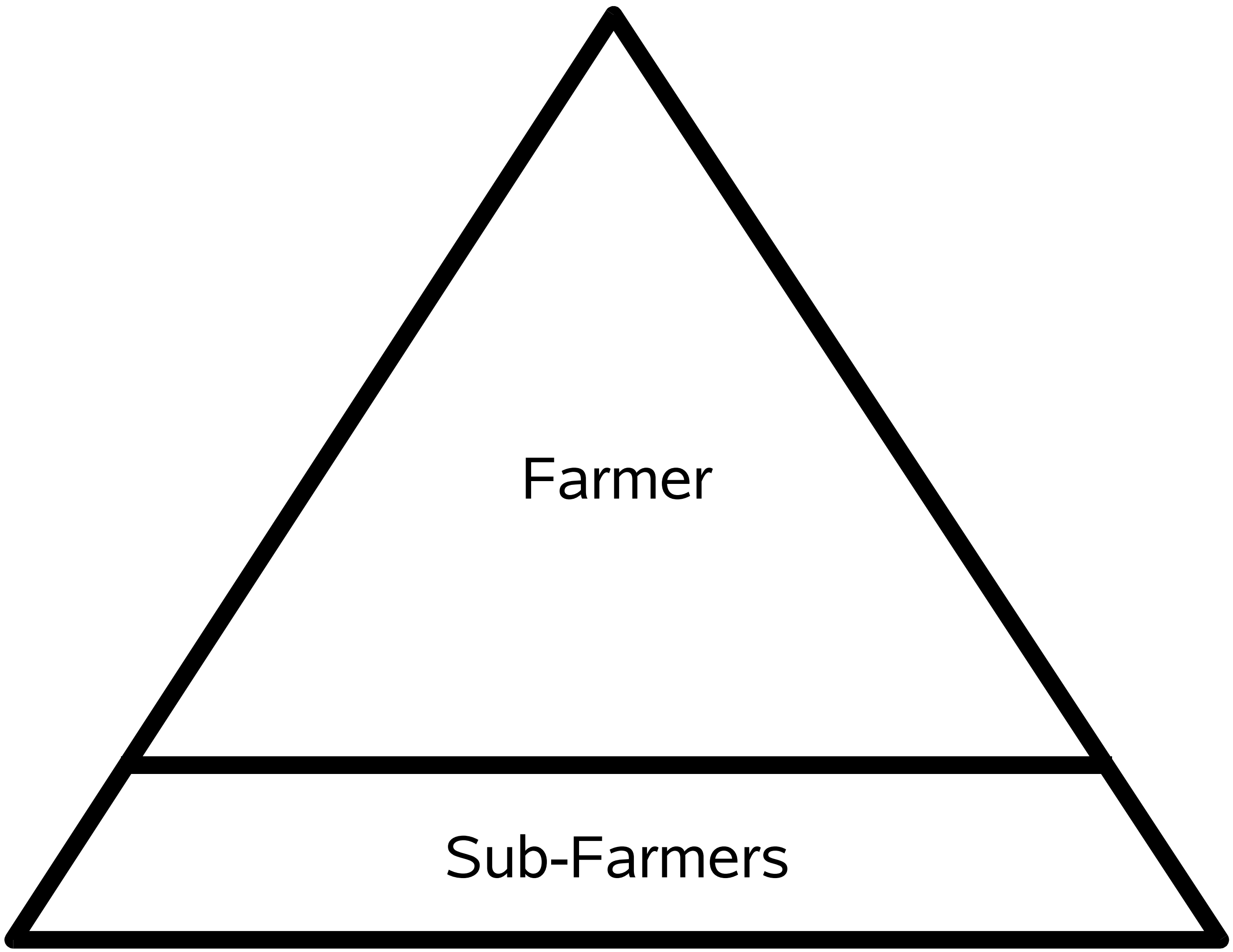
Distributed Computing

- still a lot of work to do (particularly for n>6)
- C/C++ farmer / worker model
- SALSA actor / theatre model

C++ Farmer / Worker Distribution



SALSA Actor Distribution



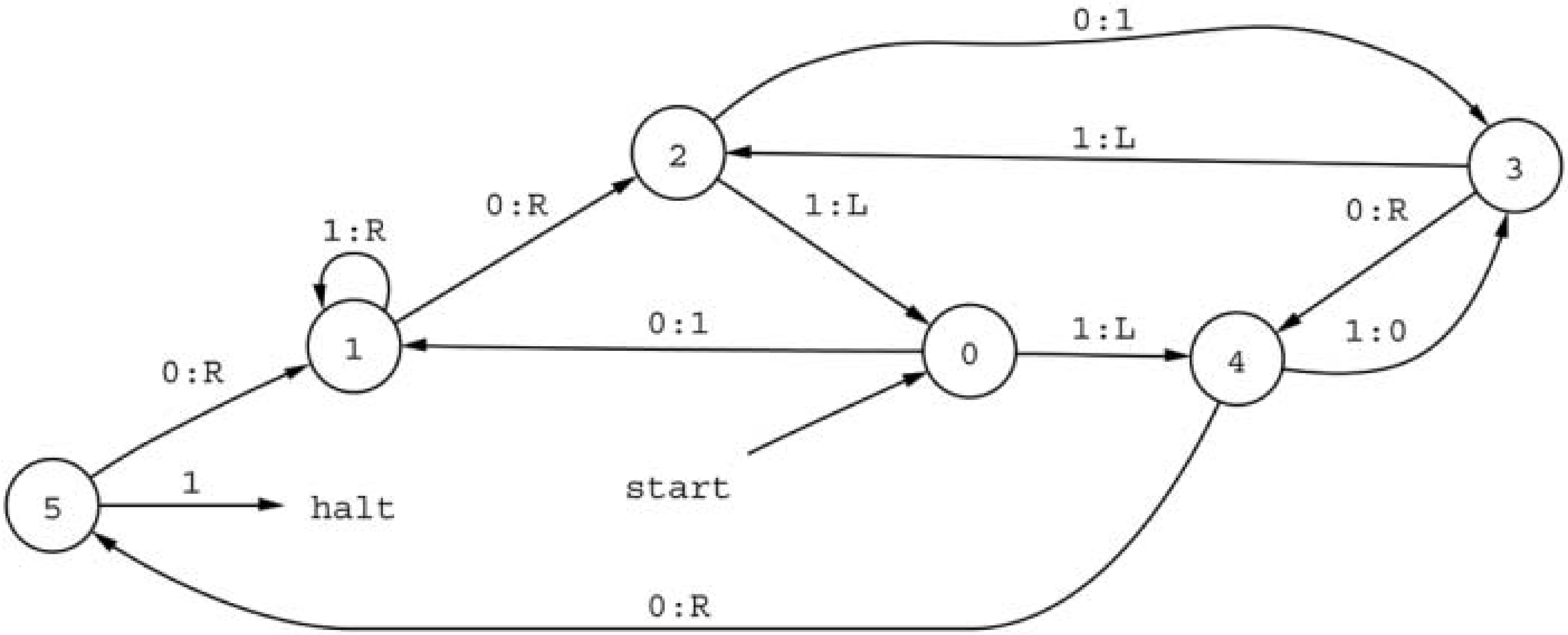
Features of Farmer / Worker

- centralised view of problem
- dynamic search-space sub-division
- compatible with optimisations
- representation and partial machines

Future of the Problem

- ultimately will always remain non-computable
- always able to get candidates
- reduction to halting problem and limits of human analysis
- Kyle's prediction: nobody will get past B(8) for a *very* long time

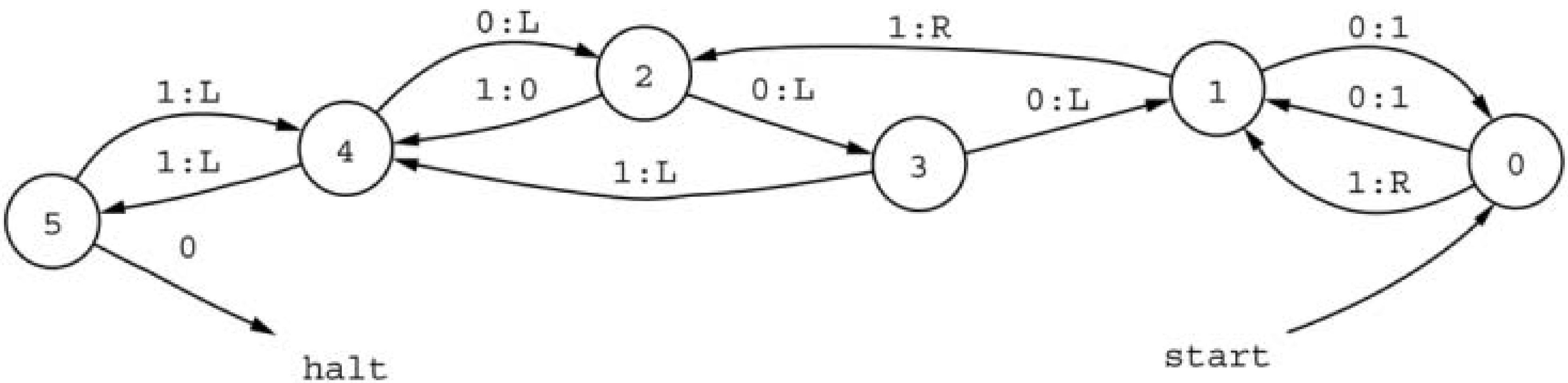
RPI B(6) Champion*



B(6)-25

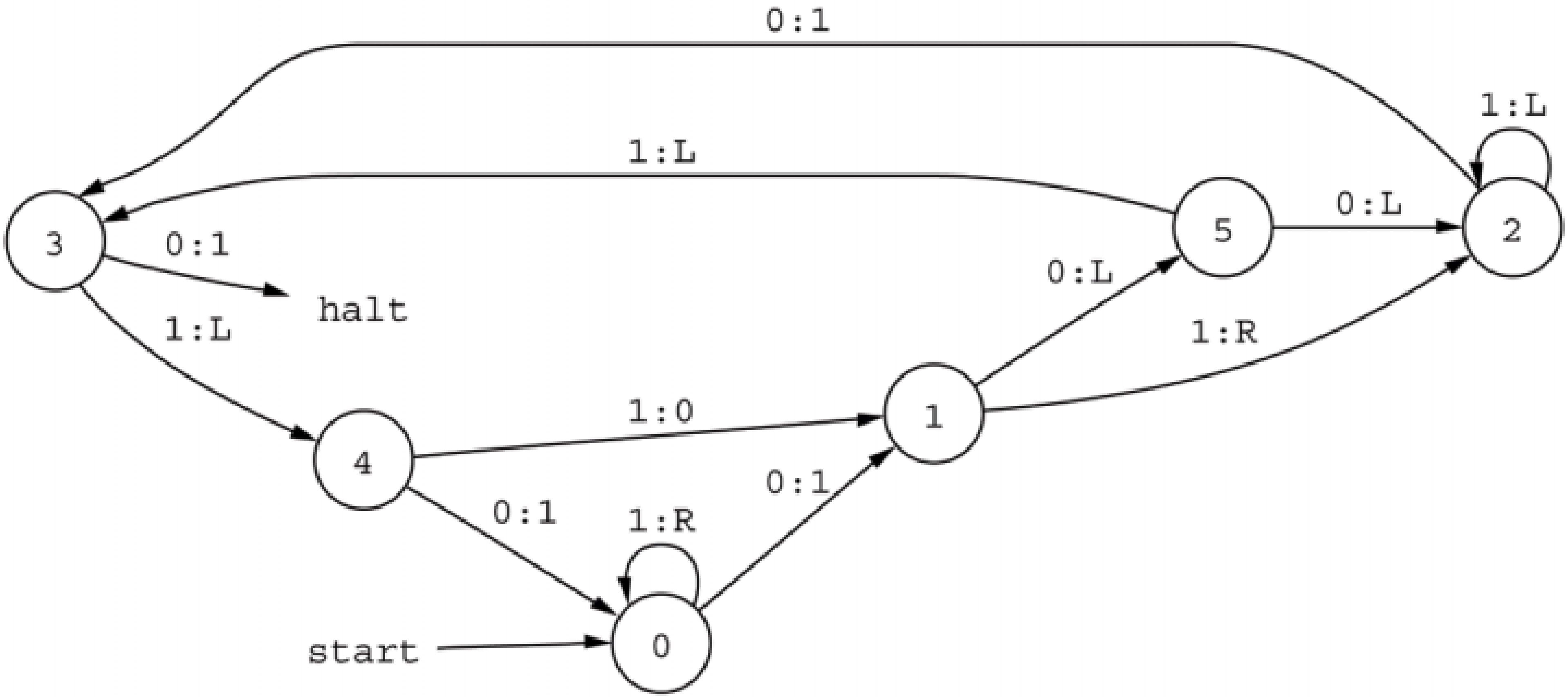
* This machine is also the world champion (and probably the theoretical B(6) champion).

We Beat the Portuguese!



P(6)-41

We Have Records for O(6) & R(6)!



R(6)-71