

Predicate Logic Proofs (all valid)

1. $\forall x (A(x) \rightarrow B(x))$ $\forall x (B(x) \rightarrow C(x))$ $\therefore \forall x (A(x) \rightarrow C(x))$	2. $\forall x (A(x) \rightarrow (B(x) \wedge C(x)))$ $\forall x (A(x) \rightarrow \neg C(x))$ $\therefore \neg A(a)$	3. $\forall x (A(x) \rightarrow \forall y B(x,y))$ $A(b)$ $\therefore \forall y B(b,y)$
4. $\forall x (B(x) \rightarrow C(x))$ $\exists y (A(y) \wedge B(y))$ $\therefore \exists z (A(z) \wedge C(z))$	5. $\neg \exists z F(z)$ $\therefore F(a) \rightarrow G(a)$	6. $\exists x A(x) \rightarrow \forall x (B(x) \rightarrow C(x))$ $A(c) \wedge B(c)$ $\therefore C(c)$
7. $\forall x (B(x) \rightarrow C(x))$ $\exists y \neg C(y)$ $\therefore \exists z \neg B(z)$	8. $K(a)$ $\forall x (K(x) \rightarrow \forall y H(y))$ $\therefore \forall x H(x)$	9. $\forall x \exists y (A(x) \rightarrow B(y))$ $\therefore \forall x F(x) \rightarrow \exists y B(y)$
10. $\exists x \forall y A(x,y)$ $\therefore \exists x A(x,a)$	11. $\forall x A(x) \rightarrow \exists y B(y)$ $\forall y \neg B(y)$ $\therefore \neg \forall z A(z)$	12. $\forall x (A(x) \rightarrow (\exists y (B(y) \wedge D(y)) \rightarrow C(x)))$ $\forall x (B(x) \rightarrow C(x))$ $\therefore \forall x (A(x) \rightarrow (B(x) \rightarrow C(x)))$
13. $\exists x R(x) \rightarrow \exists x S(x)$ $\forall y (T(y) \rightarrow R(y))$ $\therefore \exists x T(x) \rightarrow \exists z S(z)$	14. $\exists x \forall y (A(x,y) \rightarrow B(x,y))$ $\forall x \exists y \neg B(x,y)$ $\therefore \neg \forall x \forall y A(x,y)$	15. $\exists x (A(x) \wedge \forall y (B(y) \rightarrow C(x,y)))$ $\forall x \exists y (A(x) \rightarrow B(y))$ $\therefore \exists x \exists y C(x,y)$
16. $\forall x F(x) \vee \forall x \neg G(x)$ $\neg \forall x F(x)$ $\forall x (D(x) \rightarrow G(x))$ $\therefore \exists x \neg D(x)$	17. $\neg \exists x (A(x,a) \wedge \neg B(x,b))$ $\neg \exists x (C(x,c) \wedge C(b,x))$ $\forall x (B(e,x) \rightarrow C(x,f))$ $\therefore \neg (A(e,a) \wedge C(f,c))$	18. $\exists x A(x) \rightarrow \forall x (B(x) \rightarrow C(x))$ $\exists x D(x) \rightarrow \exists x \neg C(x)$ $\exists y (A(y) \wedge D(y))$ $\therefore \exists z \neg B(z)$
19. $\forall x \exists y (\neg F(x) \vee G(y))$ $\therefore \forall x F(x) \rightarrow \exists y G(y)$	20. $\forall x (A(x) \rightarrow H(x))$ $\exists x A(x) \rightarrow \neg \exists y G(y)$ $\therefore \forall x (\exists y A(y) \rightarrow \neg G(x))$	21. $\exists x (A(x) \wedge \forall y (Q(y) \rightarrow L(x,y)))$ $\forall x (A(x) \rightarrow \forall y (I(y) \rightarrow \neg L(x,y)))$ $\therefore \forall x (Q(x) \rightarrow \neg I(x))$
22. $\exists x \forall y F(x,y)$ $\forall x \forall y (F(y,x) \rightarrow G(x,y))$ $\therefore \forall y \exists x G(y,x)$	23. $\forall x \exists y F(y,x) \rightarrow \forall x \exists y G(x,y)$ $\exists x \forall y \neg G(x,y)$ $\therefore \exists y \forall x \neg F(x,y)$	24. $\exists x (F(x) \wedge \forall y (G(y) \rightarrow H(x,y)))$ $\therefore \exists x (F(x) \wedge (G(a) \rightarrow H(x,a)))$