

Short Truth Tables

Introduction to Logic

Truth Table Method

- The truth table method systematically exhausts all possible truth value combinations of the statements involved.
- In the truth-table we look for a row that reflects a certain possibility, and that will tell us the answer to whatever question we had.
 - E.g. If we want to know whether or not a certain statement is a tautology, we are interested in the possibility of that statement to be false; the statement is a tautology iff it is not possible for that statement to be false
 - For an argument: Is it possible to have all true premises and a false conclusion? If so, invalid. If not, valid.

Drawback and Room for Solution

- A drawback of the truth table method is that the number of rows grows exponentially.
- Fortunately, there is room for a solution to this problem. Since all we are interested in, is the existence of a specific combination of truth values of the statements involved, all we need to find is one example of such a case. Once we have found such a case, there is no need to exhaust all other cases.

Simple Solution: Stopping Early

- One solution to the problem of big truth tables is therefore to simply stop once you have found a row that represents the combination of truth values you are interested in.
- Thus, rather than working out a truth table column by column, you may want to do it row by row, so that you can stop as soon as you have found a row of the kind you are looking for.
- A big drawback of this approach is that if no row of the kind you are looking for exists, then you have to complete the whole truth table after all.

A More Focused Search

- Consider the following argument:

$$P \rightarrow (Q \vee R)$$

$$R \vee \neg Q$$

$$R$$

- We are interested in whether all premises can be true and the conclusion false:
 - In order for the conclusion to be false, R must be false.
 - In order for the second premise to be true while R is false, Q must be false.
 - In order for the first premise to be true while Q and R are both false, P must be false.

The Short Truth Table Method

- The Short Truth Table Method assigns truth values to the involved atomic and complex statements in order to try and obtain a certain combination of truth values (the numbers indicate the order in which the truth-values are being assigned):

$$\begin{array}{rclcl}
 P \rightarrow (Q \vee R) & R \vee \neg Q & /:\cdot & R & \\
 F_7 T_1 \quad F_5 \quad F_6 \quad F_2 & F_2 T_1 \quad T_3 F_4 & & F_1 &
 \end{array}$$

- The Short Truth Table Method thus tries to generate one row of the truth table that has the combination of truth values you are interested in.

Short Truth Table Method and Indirect Proof

- As you assign truth values to certain statements, the truth values of other statements can be forced.
- If you are forced to make a statement both true and false, then you know that the combination of truth values you are looking for does not exist:

$$\begin{array}{l} P \rightarrow (Q \rightarrow P) \quad \rightarrow \text{Contradiction (P has to be } T_2 \text{ and } F_5), \\ T_2 \ F_1 \ T_4 F_3 \ F_5 \quad \text{so the statement is a tautology!} \end{array}$$

- The short truth table method is therefore often a kind of proof by contradiction or indirect proof